Using Syntactic Possibilistic Fusion for Modeling Optimal Pessimistic Qualitative Decision

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Abstract— *This paper describes the use of syntactical data fusion to computing possibilistic qualitative decisions. More precisely qualitative possibilistic decisions can be viewed as a data fusion problem of two particular possibility distributions (or possibilistic knowledge bases): the first one representing the beliefs of an agent and the second one representing the qualitative utility. The proposed algorithm computes a pessimistic optimal decisions based on data fusion techniques. We show that the computation of optimal decisions is equivalent to computing an inconsistency degree of possibilistic bases representing the fusion of agent's beliefs and agent's preferences.*

Keywords— Data Fusion, Decision theory, Pessimistic Criteria, Possibilistic Logic.

1 Introduction

The problem of decision on uncertainty is crucial for many applications in artificial intelligence. A decision theory must provide some criteria for representing behaviors of an agent in order to take the optimal decision amongst a set of decisions. Possibility theory represents one of the theory which deals with uncertain information. This paper presents a computation of pessimistic decisions based on syntactic possibilistic fusion operations[1]. A qualitative possibilistic decision model [2] allows a gradual expression of both agent's preferences and knowledge. The preferences and the available knowledge about the world are expressed in ordinal way. In [2], the authors have proposed two qualitative criteria for ordinal decision approaches under uncertainty: the pessimistic and the optimistic decisions criteria. The first one being more cautious than the second one for computing optimal decisions. A method for computing optimal decisions, based on ATMS (Assumption-based Truth Maintenance System), has been proposed in [3]. Using the pessimistic criteria, the procedure is translated into an ATMS problem [4, 5].

Recently, in a companion paper [6] a method has been proposed for computing optimal optimistic decisions using possibilistic fusion modes. This paper is also developed in the same spirit. Indeed, qualitative possibilistic decisions can be viewed as a data fusion problem of two particular possibility distributions: the first one representing the beliefs of an agent and the second one representing the qualitative utility (or agent's preferences). The handling of possibilistic decision raises new issues and it requires additional steps such that the computation of negated possibilistic bases.
The rest of this paper is organized as follow. Section 2 gives a brief backgrounds on possibilistic logic, on qualitative decision problems under uncertainty based on possibilistic logic where agent's beliefs and the preferences are expressed by means of possibilistic bases. In Section 3, we provide a preliminary step for our work. Section 4 presents an efficient and unified way of computing pessimistic qualitative decisions based on syntactic counterpart of data fusion problem. The paper ends with some conclusions in Section 5.

2 Backgrounds

2.1 Possibilistic Logic

This section gives a brief refresher on possibilistic logic. See [7] for more details on possibilistic logic. Let $\mathcal{L}$ be a finite propositional language and Ω be the set of all propositional interpretations. $\phi, \psi, \ldots$ denote propositional formulas. For an interpretation ω and a propositional formula $\phi, \omega \models \phi$ means that ω is a model (in the sense of propositional logic) of ϕ. A possibility distribution [7, 8] π is a mapping from a set of interpretations Ω into the unit interval [0,1]. $\pi(\omega)$ represents the degree of compatibility (or consistency) of the interpretation ω with available pieces of information. Given a possibility distribution π, two dual measures are defined on the set of propositional formulas:

- The possibility (or consistency) measure of a formula ϕ, defined by:

  $$\Pi(\phi) = max\{\pi(\omega) : \omega \models \phi \text{ and } \omega \in \Omega\} \quad (1)$$

  which evaluates the extent to which ϕ is consistent with available beliefs expressed by π.

- The necessity (or certainty) measure of a formula ϕ, defined by:

  $$N(\phi) = 1 - \Pi(\neg\phi) \quad (2)$$

  which evaluates the extent to which ϕ is entailed by available beliefs.

A possibilistic knowledge base Σ is a set of weighted formulas:

$$\Sigma = \{(\phi_i, \alpha_i) : i = 1, ..., n\},$$

where ϕ_i is a propositional formula and $\alpha_i \in]0, 1]$ represents the certainty level of ϕ_i. Each piece of information (ϕ_i, α_i) of a possibilistic knowledge base can be viewed as a constraint that restricts possibility degrees associated with interpretations [7].

If an interpretation ω satisfies ϕ_i then its possibility degree is equal to 1 (ω is completely compatible or consistent with

ISBN: 978-989-95079-6-8

the belief ϕ_i), otherwise it is equal to $1-\alpha_i$ (the more ϕ_i is certain, the less ω is possible). In particular, if $\alpha_i = 1$, then any interpretation falsifying ϕ_i, is such that its possibility degree is equal to 0, namely is impossible.

More formally, the possibility distribution associated with a weighted formula (ϕ_i, α_i) is [7] $\forall \omega \in \Omega$:

$$\pi_{(\phi_i,\alpha_i)}(\omega) = \begin{cases} 1-\alpha_i & if\ \omega \not\models \phi_i \\ 1 & \text{otherwise} \end{cases} \tag{3}$$

More generally, the possibility distribution associated with a qualitative possibilistic knowledge base Σ is: $\forall \omega \in \Omega$:

$$\pi_\Sigma(\omega) = min\{\pi_{(\phi_i,\alpha_i)}(\omega) : (\phi_i, \alpha_i) \in \Sigma\}. \tag{4}$$

2.2 Possibilistic Qualitative Decision

Several works have been proposed for dealing with qualitative decision theory under uncertainty. Some approaches consider only all-or-nothing notions of utility and plausibility [9], others use in addition a preference ordering on consequences [10]. However, in many applications the knowledge bases may be pervaded with uncertainty, and the goals may not have equal priority. Dubois, Le Berre, Prade and Sabbadin [3] have enriched logical view of the decision problem by assigning levels of certainty to formulas in the knowledge bases, and levels of priority to the goals. They have proposed two syntactic approaches based on possibilistic logic, applied on two stratified logical bases that model gradual knowledge and preferences. The first one being more cautious than the second, for computing optimal decisions [3].

It has been shown in [3] that the semantics underlying the two syntactic approaches are in agreement with the two qualitative utility functions proposed in [2].

Let K be a stratified base which represents level of certainty of the knowledge, $K = \{(\phi_i, \alpha_i) : i = 1, ..., n\}$ where $\alpha_i \in S$ such that $(\alpha_i > 0)$ denotes the degree of certainty, and the ϕ_i 's are formulas in $\mathcal{L}$ where decision literals may appear.

Let P be a stratified base expressing preferences, $P = \{(\psi_i, \beta_i) : i = 1, ..., n\}$ where $\beta_i \in S$ such that $(\beta_i > 0)$ is a degree of priority, and the ψ_i's are formulas in $\mathcal{L}$ where decision literals may also appear.

Let $K_{\geq\alpha}$ denotes the set of formulas with certainty at least equal to α . Let $P_{\geq\beta}$ denotes the set of formulas with priority at least equal to β. We also recall that $K_{>\alpha}$ (with $\alpha < 1$) denotes the set of formulas with certainty strictly greater than α and $P_{>\beta}$ (with $\beta < 1$) denotes the set of formulas with priority strictly greater than β.

The certainty degrees and the priority degrees are assumed to be commensurate and assessed to the same linearly order scale S [2]. The top element of S will be denoted by 1, and the bottom element will be denoted by 0.

For any set A of formulas, $A^\wedge$ denotes the logical conjunction of the formulas in A. Let $D = \{l_i\}$ be a set of decision variables, where l_i are distinguished variables of the language $\mathcal{L}$. Let $d \subseteq D$, then the decision $d^\wedge$ is the logical conjunction of literals in the chosen subset. The variables that are not in D are state variables.

Each set of decision d induces a possibility distribution π_{K_d} in the following way [7]:

$$\pi_{K_d}(\omega) = \begin{cases} 1 \text{ if } \forall(\phi_i,\alpha_i) \in K, \omega \models \phi_i \text{ and } \omega \models d^\wedge \\ min_{(\phi_i,\alpha_i)\in K/\omega\models\neg\phi_i}(1-\alpha_i) \text{ if } \omega \models d^\wedge \\ 0 \text{ if } \omega \not\models d^\wedge \end{cases}$$

Where α_i represents the degrees of necessity of the formulas in the corresponding layers of K_d.

In other hand, the utility function μ_P associated to the preferences base P is built over Ω in a similar way:

$$\mu_P(\omega) = \begin{cases} 1 \text{ if } \forall(\psi_j,\beta_j) \in P, \omega \models \psi_j \\ min_{(\psi_j,\beta_j)\in P/\omega\models\neg\psi_j}(1-\beta_j) \end{cases}$$

where β_j represents a degree of priority of a formulas in P. Making a decision amounts to choosing a subset d of the decision set D. The objective is to rank-order decisions on the basis of K and P. The pessimistic utility function is expressed in terms of the possibility distribution π_{K_d} and the utility function μ [3]:

$$u_*(d) = min_{\omega\in\Omega} max(1-\pi_{K_d}(\omega), \mu(\omega)) \tag{5}$$

In the pessimistic case, the decision d must satisfy [1]:

$$K^\wedge_\alpha \wedge d^\wedge \vdash P^\wedge_{>(1-\alpha)} \tag{6}$$

The decision d associated with the most certain part of K entails the satisfaction of the goals, even those with low priority.

3 Syntactic Counterparts of Negated Preference Bases

This section presents a first result of this paper needed for the development of our algorithm. It consists in a characterization of a syntactic counterpart of negated possibilistic base defined by: $\forall \omega \in \Omega, \pi_{n_\Sigma}(\omega) = 1 - \pi_\Sigma(\omega)$.

Let $\Sigma = \{(\phi_i, \alpha_i) : i = 1, ..n\}$ be a possibilistic base. We assume that: $\alpha_0 = 0 < \alpha_1 < ... < \alpha_n$. The following definition gives the possibilistic knowledge base associated with the negation of Σ.

Definition 1. The negated base of $\Sigma = \{(\phi_i, \alpha_i) : i = 1, ..., n\}$ is a possibilistic base, denoted by n_Σ, and defined by:

$$n_\Sigma = \{(d_i, 1-\alpha_i) : i = 1, ..., n\} \cup \{(\bot, 1-\alpha_n)\}$$

where $d_i = \neg\phi_i \vee \neg\phi_{i+1} \vee ... \vee \neg\phi_n$.

Example 1. Let $\Sigma = \{(\neg a \vee b, 0.3), (b \vee \neg c, 0.6)\}$. Then, $n_\Sigma = \{((a \wedge \neg b) \vee (\neg b \wedge c), 0.7), (\neg b \wedge c, 0.4)\}$

The following proposition shows that n_Σ is indeed encodes the negation of Σ:

Proposition 1. Let $\Sigma = \{(\phi_i, \alpha_i) : i = 1, ..., n\}$ be a preference base, and n_Σ its negated base obtained using Definition 1. Let π_Σ and π_{n_Σ} be the utility distributions associated with Σ and n_Σ respectively. Then:

$$\forall \omega \in \Omega, \pi_\Sigma(\omega) = 1 - \pi_{n_\Sigma}(\omega).$$

Proof. Recall first that when $\pi_\Sigma(\omega) = \alpha$ (resp $\pi_{n_\Sigma} = \beta$) then ω falsifies formulas of Σ (resp of n_Σ) having a weight equal to α (resp β), but satisfies all formulas of Σ (resp n_Σ) having a weight strictly larger than α (resp β). We distinguish 3 cases:

ISBN: 978-989-95079-6-8

1. $\pi_\Sigma(\omega) = 1$, which means that $\forall(\phi_i, \alpha_i) \in \Sigma, \omega \models \phi_i$. This means that ω falsifies all formulas of n_Σ, and in particular the highest formula α_1, namely $\omega \not\models \alpha_1$. Hence, $\pi_{n_\Sigma}(\omega) = 0 = 1 - \pi_\Sigma(\omega)$.
2. $\pi_\Sigma(\omega) = \alpha_n$. This means that ω satisfies all formulas in Σ except α_n. This also means that ω satisfies all formulas d_i (since d_i is a disjunction of $\neg\phi_n$ and others terms). The only formula falsified by $\omega \in n_\Sigma$ is $(\bot, 1 - \alpha_n)$. Hence, $\pi_{n_\Sigma}(\omega) = 1 - (1 - \alpha_n) = \alpha_n = \pi_\Sigma(\omega)$.
3. $\pi_\Sigma(\omega) = \alpha_j$ with $j < n$. This means that $\omega \not\models \phi_j$. Hence, $\forall i \leq j, \omega \models \phi_j$ (since d_j contains $\neg\phi_j$). $\mu_P(\omega) = \alpha_j$ also means that $\forall k > j, \omega \models \phi_k$, namely $\omega \models \phi_n \wedge ... \wedge \phi_{j+1}$. Hence, $\omega \not\models \neg\phi_n \vee ... \vee \phi_{j+1}$. Namely, $\omega \not\models d_{j+1}$. Therefore: $\pi_{n_\Sigma}(\omega) = 1 - (1 - \alpha_{(j+1)-1}) = \alpha_j = \pi_\Sigma(\omega)$.

The obtained base n_Σ must be put in a clausal form to get C_{n_Σ}.

Example 2. Let n_Σ be the base obtained in example 1. The clausal form of n_Σ is then: $C_{n_\Sigma} = \{(c, 0.4), (a \vee \neg b, 0.7), (a \vee c, 0.7), (\neg b, 0,7), (\neg b \vee c, 0.7)\}$

4 Computation of Pessimistic Decision

In this section, we propose an algorithm for computing qualitative optimal decision in the pessimistic case. The knowledge base K can be the result of merging several knowledge bases $K_1, K_2, ..., K_n$. The possibilistic distribution associated to K is then obtained by merging possibility distributions associated with $K_1, , K_2, ..., K_n$ using some merging operator /oplus, namely: $\pi_K = \oplus(\pi_{K1}, ..., \pi_{Kn})$. When $\oplus = min$, it has been shown that $K = K_1 \cup ...K_n$. If $\oplus = max$, then $K = \{(\phi_1 \vee ...\phi_n, min(\alpha_1, ..., \alpha_n)) : (\phi_1, \alpha_1) \in K_1, ..., (\phi_n, \alpha_n) \in K_n\}$. For syntactic counterparts of more general operator, see [1]. Similarly, P can be also the result of merging several preference bases.
Now, once K and P are fixed, we propose a syntactic computation of optimal decisions. We recall that an optimal pessimistic decision d maximizing $u_*(d)$ is such that:

$$u_*(d) = min_{\omega \in \Omega} max(1 - \pi_{K_d}(\omega), \mu(\omega)) \quad (7)$$

From equation (6), a first way to syntactically compute optimal decisions is first to compute the counterpart of $1 - \pi_{K_d}$, then compute the counterpart of $max(1 - \pi_{K_d}(\omega), \mu(\omega))$ and lastly compute $u_*(d)$. This approach is not interesting since first in general Σ is more important than P and computation of $max(1 - \pi_{Kd}(\omega), \mu(\omega))$ is more expensive than the computation of the minimum. We propose to explore another possibility by noting that equation (6) is equivalent to:

$$u_*(d) = 1 - max_{\omega \in \Omega} min(\pi_{K_d}(\omega), 1 - \mu(\omega)) \quad (8)$$

Besides, the syntactic counterpart of $min(\pi_1(\omega), \pi_2(\omega))$ is the possibilistic base $\Sigma_{min} = \Sigma_1 \cup \Sigma_2$ [1].
Thus, combining these results, the corresponding base Σ_{min} associated to $min(\pi_{K_d}(\omega), 1 - \mu(\omega))$ is the possibilistic base $K \cup n_P \cup \{(d, 1)\}$, such that n_P is the possibilistic base corresponding to the utility function $1 - \mu(\omega)$. We recall that:

$$u_*(d) = 1 - max_{\omega \in \Omega}\{min(\pi_{K_d(\omega)}, 1 - \mu(\omega))\} \quad (9)$$

Lemma 1. Let n_P be the negated based, obtained using Definition 1, of the preferences base P . Then, the syntactic counterparts of $min(\pi_{K_d(\omega)}, 1 - \mu(\omega))$ is $K \cup n_P$.

On the other hand, the inconsistency degree $Inc(K \cup C_{n_P})$ of a possibilistic base $K \cup C_{n_P}$, where C_{n_P} represents the conjunctive form of the base n_P, is defined as follow [7]:

$$Inc(K \cup C_{n_P}) = 1 - max\{\pi_{K_d}(\omega)\}$$

Lemma 2. The pessimistic utility function associated to decision d is :

$$u_*(d) = Inc(\Sigma_{min})$$

Where $Inc(\Sigma_{min})$ represents the inconsistency degree of the base $K \cup C_{n_P} \cup \{(d, 1)\}$.
Then, the computation of optimal pessimistic decisions is obtained using the Algorithm 1.
The computation of inconsistency degree is performed by a call to the function $incons(B \cup \{(\neg\phi, 1)\}, Inc, bool)$. This function, given by the Algorithm 2, has three parameters: a stratified knowledge base, an integer representing current inconsistency degree and a boolean variable.

Algorithm 1 Computation of optimal pessimistic decision

```
Input: K knowledge base
       C_{n_P} conjunctive form of n_p
       N number of decision variables
       D = {d_{i∈[1,n]}} set of decisions
Output: decision
Begin
i ← 1
max ← 0
inc ← 1
for i = 1 to N do
  incons(K ∪ C_{n_P} ∪ {(d_i, 1)}, inc, bool)
  if (bool=true) then
    if (inc > max) then
      max ← inc
      decision ← {d_i}
    else if inc = max then
      decision ← decision ∪ {d_i}
    end if
  end if
end for
RETURN (decision)
End
```

The base $B^*_{\geq\alpha_r}$ is defined as the classical projection of the α_cut $B_{\geq\alpha_r}$. $B^*_{\geq\alpha_r} = \{\phi : (\phi, \beta) \in B, \beta \geq \alpha_r\}$.
The function incons is on the extension of dichotomous algorithm for computing inconsistency proposed in [11] on which some adaptations have been made in order to compute optimal decision. Indeed, one the inconsistency degree of $K \cup C_{n_P} \cup \{(d_i, 1)\}$ can be greater than the inconsistency degree of $K \cup C_{n_P} \cup \{(d_{j\in[1,i-1]}, 1)\}$, the algorithm stops.
In terms of complexity, the proposed algorithm is based on

ISBN: 978-989-95079-6-8

an inconsistency degree computation which is known to be NP-hard and requires in the worst case $[log_2 m]$ satisfiability checks, where m is the number of different valuations involved in $K \cup C_{n_P} \cup \{(d_i, 1)\}$ using any prouver for the propositional satisfiability problem SAT.

Algorithm 2 Function $incons(B \cup \{(\neg\phi, 1)\}, inc, bool)$

Input B:stratified base
    ϕ:weighted formula
    n: number of strates in base B
Output inc: inconsistency degree
    bool:boolean
Begin
$l \leftarrow 0$ {initially pointed on the last strate of the base}
$u \leftarrow m$ {initially pointed on first strate of the base}
$bool \leftarrow true$
while $((l < u)$ and $(bool = true))$ **do**
  $r \leftarrow [(l + u)/2]$ {pointer uses for dichotomy}
  if ($B^*_{\geq\alpha_r} \wedge \neg\phi$ consistent) **then**
    $u \leftarrow r - 1$ {check the inconsistency in the most big base}
  else
    if $(inc \geq \alpha_r)$ **then**
      $l \leftarrow r$
    else
      $bool \leftarrow false$
    end if
  end if
end while
if $(\alpha_r \geq inc)$ **then**
  $bool \leftarrow false$
end if
if (bool = true) **then**
  $inc \leftarrow \alpha_r$
end if
RETURN(inc,bool)
End

Example 3. Let us exemplify the algorithm on an example initially proposed in [12] and reused in [2]. The example is about taking an umbrella or not, knowing that the sky is cloudy.
The literals of the language are:

- It rains: r
- Being wet: w
- Taking an umbrella: um
- The sky is cloudy: c

The stratified knowledge base is $K = \{(\neg um \vee \neg w, 1), (\neg r \vee um \vee w, 1), (r \vee \neg w, 1), (c, 1), (\neg c \vee r, 0.6)\}$.
The stratified preference base is $P = \{(\neg um, 0.2), (\neg w, 1)\}$. We do not like to take an umbrella, but it is more important to be dry.
The set of decisions is $D = \{um, \neg um\}$, i.e taking an umbrella or not.

The preliminary step consists to compute the conjunctive normal form C_{n_P} of the negated form n_P of the preference base P. Then, the algorithm applies as follow:

- initially, $i \leftarrow 1$, $inc \leftarrow 1$, max $\leftarrow 0$ and $d_1 \leftarrow um$. We have to call to the function $incons(K \cup C_{nP} \cup \{(um, 1)\}, inc, bool)$. This function return inc=0.1 and bool=true. In this case, $inc > max$, so $decision \leftarrow \{um\}$ and $max \leftarrow inc$.
- In the next step, $i \leftarrow 2$ and $d_2 \leftarrow \neg um$. The call of the function $incons(K \cup C_{nP} \cup \{(\neg um, 1)\}, inc, bool)$ returns $inc = 0.1$ and bool is true. As inc=max, the set of optimal decisions do not change.

Thus, the best pessimistic solution of the umbrella problem is to take an umbrella.

5 Conclusion

The main contribution of this paper is a proposition of a new approach to compute a qualitative possibilistic pessimistic decision problem. This problem is viewed as the one of computing inconsistency degrees of particular bases in the framework of possibilistic logic. The application exploits the syntactic counterparts of data fusion techniques.
Our approach is an alternative to the ATMS-based solution, proposed in [3] and it avoids the use of the ATMS to compute the pessimistic optimal qualitative decision which is known to be not efficient when it deals with an important number of variables. Then, in the pessimistic case, the knowledge base and the preferences base are fused and the decision problem is translated into the computation inconsistency degree of a specific base. This process leads to use a logical technique more adapted than the ATMS one.

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ISBN: 978-989-95079-6-8

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ISBN: 978-989-95079-6-8