Interactive Fuzzy Modeling by Evolutionary Multiobjective Optimization with User Preference

Yusuke Nojima, Hisao Ishibuchi

Department of Computer Science and Intelligent Systems, Osaka Prefecture University
1-1 Gakuen-cho, Naka-ku, Sakai, Osaka 599-8531, Japan
Email: {nojima, hisaoi}@cs.osakafu-u.ac.jp

Abstract— *One of the new trends in genetic fuzzy systems (GFS) is the use of evolutionary multiobjective optimization (EMO) algorithms. This is because EMO algorithms can easily handle two conflicting objectives (i.e., accuracy maximization and complexity minimization) when we design accurate and compact fuzzy rule-based systems from numerical data. Since the main advantage of fuzzy rule-based systems compared with other non-linear ones is their linguistic interpretability, the design of fuzzy rule-based systems can be viewed as linguistic data mining from numerical data. From the data mining point of view, the required knowledge strongly depends on its user. That is, the interpretability of fuzzy rule-based systems should be evaluated by taking into account the user's preference. Although there exist a number of interpretability measures in the literature, users usually do not know which measure represents their preference beforehand. In this paper, we propose interactive fuzzy modeling by evolutionary multiobjective optimization with user's preference. User's preference is represented by several satisfaction level functions which can be interactively modified by the user. The user's preference is used as one of multiple objectives in an EMO algorithm. As a case study, we apply our approach to real world time-series data of land price movements in Japan and demonstrate a user interface of our approach.*

Keywords— evolutionary multiobjective optimization, fuzzy modelling, interactive evolutionary computation, user preference.

1 Introduction

There are two major goals in the design of fuzzy rule-based systems: accuracy maximization and complexity minimization. Since the mid-1990s, a large number of approaches have been proposed for improving the accuracy of fuzzy rule-based systems while maintaining their interpretability [1-22]. Genetic algorithms have been frequently used under the name of genetic fuzzy systems (GFS) [23-25]. We can easily handle two conflicting objectives: accuracy maximization and complexity minimization by the weighted sum of them or multiobjective formulations using evolutionary multiobjective optimization (EMO) algorithms [26-28]. One of the hottest issues in GFS is how to measure the interpretability of fuzzy rule-based systems [29-34]. A number of interpretability measures have been already proposed and implemented in GFS. Interpretability is, however, very subjective for users. Let us assume the following two rule sets.

[Fuzzy rule-based system A]

If x_1 is *big* then y is 10,
If x_3 is *small* then y is 5,
If x_2 is *very small* then y is 2,
If x_3 is *big* then y is 11.

[Fuzzy rule-based system B]

If x_1 is *big* and x_2 is *big* and x_3 is *small* then $y=x_1+5x_2+9$,
If x_1 is *very big* and x_2 is *very small* then $y = 2x_2+2$,
If x_1 is *very small* and x_2 is *small* and x_3 is *big* then y=12.

If we assume that a rule set with a small number of rules is interpretable, the fuzzy rule-based system B is more interpretable. However, the fuzzy rule-based system A seems to be more interpretable with respect to the rule length and the rule type. From this observation, we can say that the interpretability is totally dependent on user's preference (see [35] for more detailed discussions). There is another problem on interpretability. Users usually know which is interpretable for them only after comparing among some alternative fuzzy rule-based systems. That is, an interactive optimization process must be needed for the users.

In our former studies [36, 37], we have proposed an interactive genetic fuzzy rule selection method for pattern classification problems. A preference function is used as one objective function in an EMO algorithm. The preference function is composed of several satisfaction level functions. The reason why we used several satisfaction level functions is that users don't know appropriate criteria and their priorities among them beforehand. These satisfaction level functions are interactively modified during the evolution under the framework of interactive evolutionary computation [38-41]. In this paper, we apply this idea to a fuzzy modeling problem. We deal with time-series data of land price movements and demonstrate the prototype of a user interface.

This paper is organized as follows: Section 2 explains fuzzy modeling and its interactive optimization process. Section 3 explains the tackled problem and demonstrates the effectiveness of our method with a prototype of our user interface. Section 4 concludes this paper.

2 Interactive Fuzzy Modeling

2.1 Fuzzy Modeling

In this paper, for an n-input and single-output nonlinear function $y = y(\mathbf{x})$, we use the following fuzzy if-then rules:

$$\text{Rule } R_k : \text{If } x_1 \text{ is } A_{k1} \text{ and ... and } x_n \text{ is } A_{kn} \text{ then } y \text{ is } B_k,\ \ k = 1, 2, ..., N, \tag{1}$$

where x_i is the i-th input variable of an n-dimensional input vector $\mathbf{x} = (x_1, \ldots, x_n)$, y is an output variable, k is a rule index, A_{ki} is an antecedent linguistic label (e.g., *small* and *large*) for

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x_i, B_k is a consequent linguistic value, and N is the total number of fuzzy if-then rules.

The following fuzzy reasoning method has been frequently used in fuzzy rule-based systems since its first proposal in a neuro-fuzzy system [42]:

$$\hat{y}(\mathbf{x}) = \frac{\sum_{k=1}^{N} \mu_k(\mathbf{x}) \cdot b_k}{\sum_{k=1}^{N} \mu_k(\mathbf{x})}, \tag{2}$$

where $\mu_k(\mathbf{x})$ is the compatibility grade of the fuzzy if-then rule R_k with the input vector $\mathbf{x}$, and b_k is a representative real number of the consequent linguistic value B_k. The compatibility grade $\mu_k(\mathbf{x})$ is usually calculated by the product operation as

$$\mu_k(\mathbf{x}) = \mu_{k1}(x_1) \times \cdots \times \mu_{kn}(x_n), \tag{3}$$

where $\mu_{ki}(\cdot)$ is the membership function of the antecedent linguistic value A_{ki}. The representative real number b_k can be viewed as a result of the defuzzification of the consequent linguistic value B_k.

The fuzzy reasoning method in (2) can be viewed as a simplified version of the Takagi-Sugeno (TS) model where a linear function is used in the consequent part of each fuzzy if-then rule. The simplified fuzzy reasoning method in (2) has several advantages. For example, its reasoning mechanism is very simple, and it is suitable for gradient-based learning algorithms.

Since we use multiple granularities of fuzzy sets for an input vector, sometime the effect of specific rules becomes lower due to general rules. For giving specific rules more weight, we use an idea of inclusion-based fuzzy reasoning [43]. We extend the inclusion relation $R_A \subset R_B$ in [43]. When only the two rules R_k and R_q with the relation $|\mathbf{A}_k| > |\mathbf{A}_q|$, are compatible with the input vector $\mathbf{x}$, the specific rule R_q is mainly used in fuzzy reasoning. That is, the weight of the general rule R_k is discounted. Our idea is to determine the amount of the discount for R_k using the compatibility grade $\mu_q(\mathbf{x})$ of the specific rule R_q with the input vector $\mathbf{x}$. More specifically, the weight of R_k is defined as $(1-\mu_q(\mathbf{x}))$. When the specific rule R_q is fully compatible with the input vector $\mathbf{x}$, the weight of the general rule R_k is zero. This means that R_k has no effect on the calculation of the estimated output value $\hat{y}(\mathbf{x})$. On the other hand, when the compatibility grade of R_q with $\mathbf{x}$ is very small, the amount of the discount for R_k is also very small. In this case, R_k has almost the same weight as R_q. Since the general rule R_k may include multiple rules, its weight is defined as

$$w(R_k, \mathbf{x}) = \prod_{\substack{q=1 \\ |\mathbf{A}_q| < |\mathbf{A}_k|}}^{N} (1 - \mu_q(\mathbf{x})). \tag{4}$$

When there are no compatible fuzzy if-then rule smaller than R_k, $w(R_k, \mathbf{x})$ is specified as $w(R_k, \mathbf{x}) = 1$ because the weight of R_k should not be discounted in this case. It should be noted that the weight of each rule depends on the compatibility grades of other rules with the input vector $\mathbf{x}$. This means that the weight is context-dependent. Different weights are assigned to the same rule for different input vectors. Moreover, the same rule may have different weights for the same input vector in different rule bases because the weight of each rule depends on other rules.

Using the rule weight $w(R_k, \mathbf{x})$ of each fuzzy if-then rule R_k, our inclusion-based fuzzy reasoning method is written as

$$\hat{y}(\mathbf{x}) = \frac{\sum_{k=1}^{N} w(R_k, \mathbf{x}) \cdot \mu_k(\mathbf{x}) \cdot b_k}{\sum_{k=1}^{N} w(R_k, \mathbf{x}) \cdot \mu_k(\mathbf{x})}. \tag{5}$$

2.2 *Multiobjective Fuzzy Rule Selection for Modeling*

We use a simple two-stage method for designing rule sets. In the first phase, a large number of candidate rules are generated from the possible combinations of membership functions. The consequent real number is specified as the weighted average of output values of compatible input-output pairs as

$$b_k = \frac{\sum_{p=1}^{m} \mu_{\mathbf{A}_k}(\mathbf{x}_p) \cdot y_p}{\sum_{p=1}^{m} \mu_{\mathbf{A}_k}(\mathbf{x}_p)}, \tag{6}$$

where $\mu_{\mathbf{A}_k}(\mathbf{x}_p)$ is the compatibility grade of the input vector $\mathbf{x}_p$ with the antecedent part $\mathbf{A}_k$ of the linguistic rule R_k.

In the second phase of our rule selection, a number of fuzzy rule sets are selected by a multiobjective genetic algorithm. Any subset S of the N candidate rules can be represented by a binary string of length N as

$$S = s_1 s_2 \cdots s_N, \tag{7}$$

where s_k=1 and s_k=0 represent the inclusion of the k-th candidate rule R_k in S and the exclusion of R_k from S.

Each fuzzy rule set S is evaluated by the three objectives:

$f_1(S)$: the total square error by S,

$f_2(S)$: the number of selected fuzzy rules in S,

$f_3(S)$: the overall user preference for S.

The first and second objectives have been frequently used and correspond to accuracy maximization and complexity minimization, respectively. The first objective is calculated by,

$$f_1(S) = \sum_{p=1}^{m} (y_p - \hat{y}_p(\mathbf{x}_p))^2. \tag{8}$$

The third objective $f_3(S)$ is the newly proposed objective in this paper. We explain it in the next subsection. The problem formulation of multiobjective genetic fuzzy rule selection is written as

$$\text{Minimize } f_1(S) \text{ and } f_2(S), \text{ and maximize } f_3(S). \tag{9}$$

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We use NSGA-II of Deb et al. [27] to search for a number of non-dominated fuzzy rule-based systems with respect to these three objectives. In this paper, uniform crossover and biased bit-flip mutation are used in NSGA-II. The biased mutation is that a larger probability is assigned to the mutation from 1 to 0 than that from 0 to 1.

Figure 1 shows the whole procedure of the proposed method. We specify an interval (i.e., the number of generations) for internal evaluations. During this interval, the satisfaction level functions are not changed. After the interval, the user checks some of non-dominated rule sets and modifies the satisfaction level functions. Then another internal evaluation process starts. By repeating this interactive process, the user can specify the own preference and find the rule set with the high user preference value.

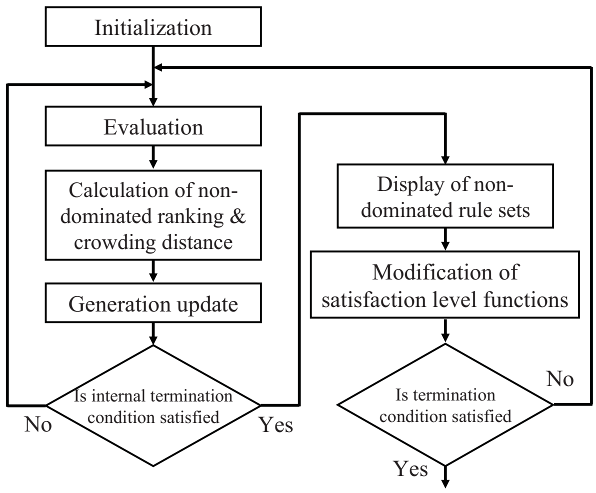

Figure 1: The whole procedure of the proposed method.

2.3 *Preference Function*

In our former study [36, 37], we have proposed a preference function composed of several satisfaction level functions. The inputs for the satisfaction level functions are criteria on the accuracy and interpretability of fuzzy rule sets. Each satisfaction level function is represented by a trapezoidal function in Fig. 2. $g^r(S)$ is the value of r-th criterion for the rule set S. $u^r(g^r(S))$ is the output of r-th satisfaction level function. Users specify the preference and priority for each criterion by moving the point B in Fig. 2. That is, the u_x and u_y of the point B mean the preference and priority for the criterion, respectively. The u_x can be also regarded as the maximum criterion level.

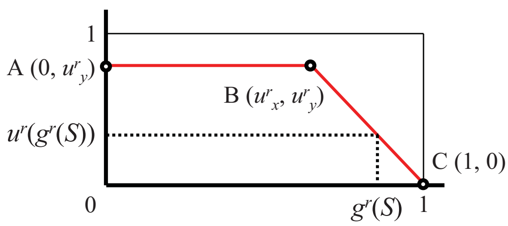

Figure 2: A trapezoidal function for representing satisfaction level functions of each criterion.

The satisfaction level function can be viewed as the requirement level. During evolution, users can modify the satisfaction level function according to the temporally obtained non-dominated rule sets.

The third objective function $f_3(S)$ for an overall user's preference is calculated by

$$f_3(S) = \sum_{r=1}^{N_C} u^r\left(g^r(S)\right), \tag{10}$$

where N_c is the number of criteria.

3 Case Study

3.1 *Problem Description*

We apply our proposed method to a simple fuzzy modelling problem for time-series data. The data we used in this paper is the land price movements of the three major metropolitan areas in Japan available from Ministry of Land, Infrastructure, Transport and Tourism webpage (http://www.mlit.go.jp/index_e.html). The data includes the land price movements from 1980 to 2000. In this period, the bubble economy was a big problem: the increased demand for office buildings in city centres due to internationalization and informatization.

The data is composed of 63 pairs of two inputs (i.e., *area* and *year*) and one output (*change of land price*). For simplicity, we normalized the input attributes into [0, 1]x[0, 1] space. We used seven categorical values (all possible combinations) for *area* attribute. For year attribute, we used 48 fuzzy membership functions shown in Fig. 4 and *don't care* condition. Each peak of triangular membership functions corresponds to one of years. These partitions could be understandable (e.g., 90's, Mid of 80's, around 1988).

From this data, 343 fuzzy if-then rules were generated as candidate rules. Thus, the search space is 2^{343}.

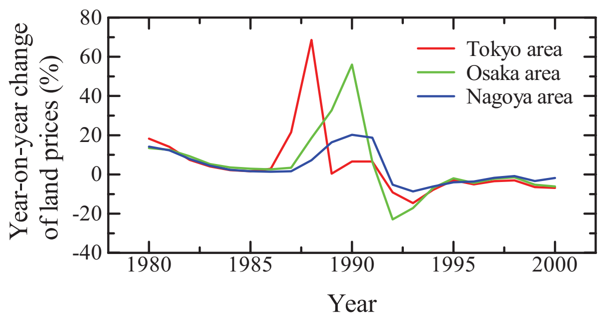

Figure 3: Year-on-year change of land prices of three major metropolitan areas in Japan from 1980 to 2000.

3.2 *Criteria for Representing User's Preference*

There are a lot of interpretability measures in the literature. In this case study, we used four simple criteria like:

- Maximum square error by S,
- Overlap among antecedent sets in S,
- Total square error by S,
- The number of fuzzy rules in S.

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The maximum square error is calculated by

$$g^1(S) = \max_{p=1,\ldots,m} (y_p - \hat{y}_p(\mathbf{x}_p))^2 . \qquad (11)$$

When a user gives a high priority to this criterion, the larger changes could be fitted by a fuzzy rule set.

We normalized each value of four criteria within the valid ranges based on the distribution values in the pre-simulation without user's preference. The reason why we used *the total square error* and *the number of fuzzy rules* as the criteria for user's preference is to reduce the search space based on the user's preference.

The choice of interpretability measures is future research issues. The correlation among measures must be examined.

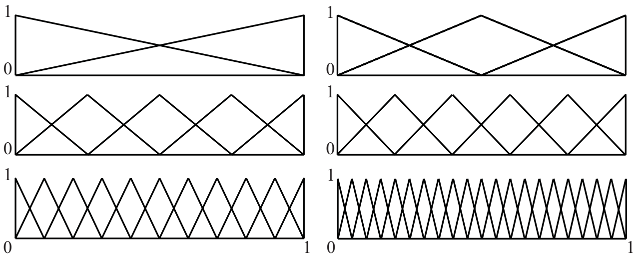

Figure 4: Fuzzy partitions with different granularities for *Year* attribute.

3.3 Prototype of User Interface

Figure 5 shows the prototype of our user interface. There are two windows. The left one represents the actual land price movement and the inferred land price movement calculated by the chosen fuzzy rule set.

At the middle of the left window, there are two graphs for representing non-dominated rule sets in terms of "the total square error and the number of rules" and "the total square error and the user preference value". Red open plots represent non-dominated rule sets. Blue closed plot means the chosen rule set. The above inferred land price movement corresponds to the chosen rule set. Users can choose one of the non-dominated rule sets by clicking any plot in the graphs. The right CUI window shows the rules in the chosen rule set.

At the bottom of the left window, there are four satisfaction level functions. Users can change the shapes of these functions by moving each point B in Fig. 2.

The button "Evolve" is a trigger to start internal evaluations. In this paper, we specified the number of generations for internal evaluations as 100.

3.4 Some Results

When a user specified the satisfaction level functions for the maximum error and the number of rules as in Fig. 6, the user obtained a rule set with a small number of rules. The rule set seems to represent the original characteristics of the data. From Table 1, we can see that there are some general rules and specific rules in the fuzzy rule set. The value in parentheses in Table 1 represents the range of 0.5-level set of used membership function. For example, "1990 [1]" means the smallest partition in which the peak is 1990.

Table 1: Obtained rule set in Fig. 6.

Area	Year	Change %
Osaka	1990 [1]	56.1
Osaka	1992 [1]	-22.9
Tokyo	1988 [1]	68.6
Tokyo & Osaka	1993 [1]	-15.9
Osaka & Nagoya	1990 [2]	28.4
Tokyo & Osaka & Nagoya	1988 [4]	17.2
Tokyo & Osaka & Nagoya	-	4.3

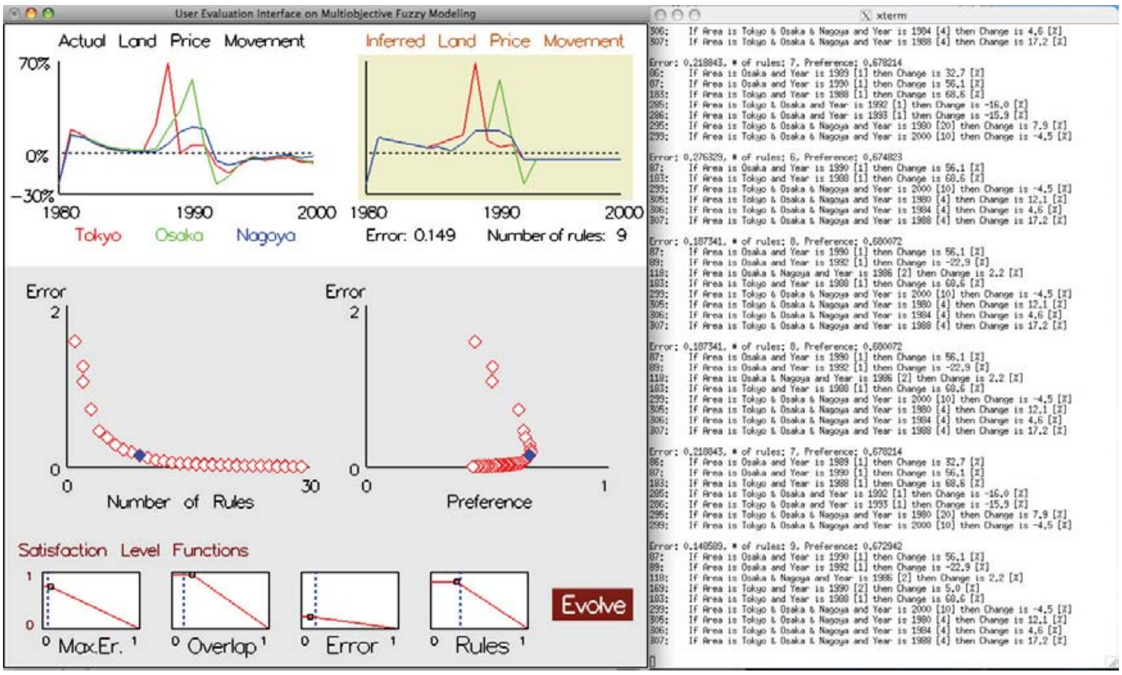

Figure 5: Prototype of user interface for interactive fuzzy modelling.

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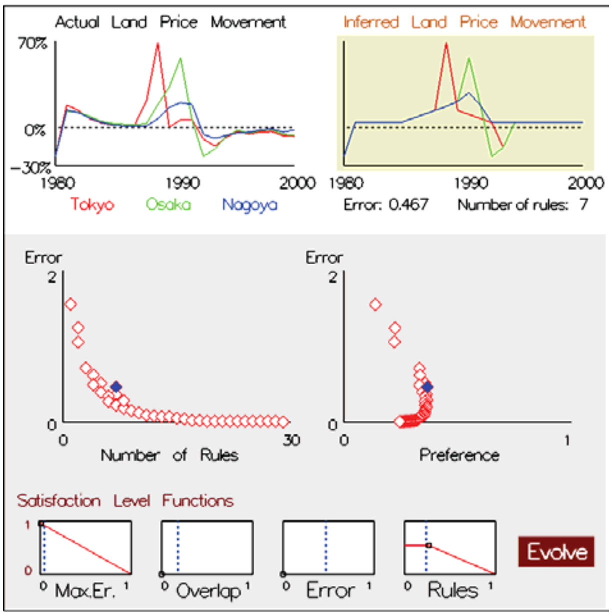

Figure 6: Example 1.

When a user gave a high priority to "Overlap among antecedent sets in S" as in Fig. 7, the user obtained a rule set with only one general rule and six specific rules (see Table 2).

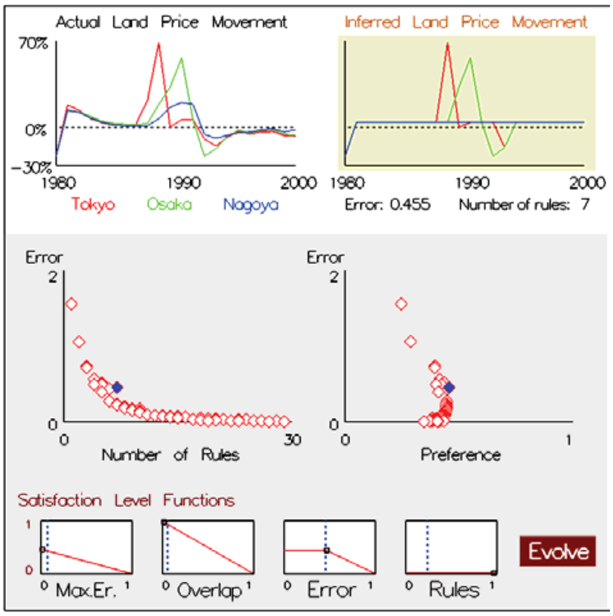

Figure 7: Example 2.

Table 2: Obtained rule set in Fig. 7.

Area	Year	Change %
Osaka	1989 [1]	32.7
Osaka	1990 [1]	56.1
Osaka	1992 [1]	-22.9
Tokyo	1988 [1]	68.6
Tokyo	1989 [1]	0.4
Tokyo & Osaka	1993 [1]	-15.9
Tokyo & Osaka & Nagoya	-	4.3

When a user gave high priorities to accuracy criteria (i.e., maximum error and total square error), a very accurate rule set was obtained as in Fig. 8.

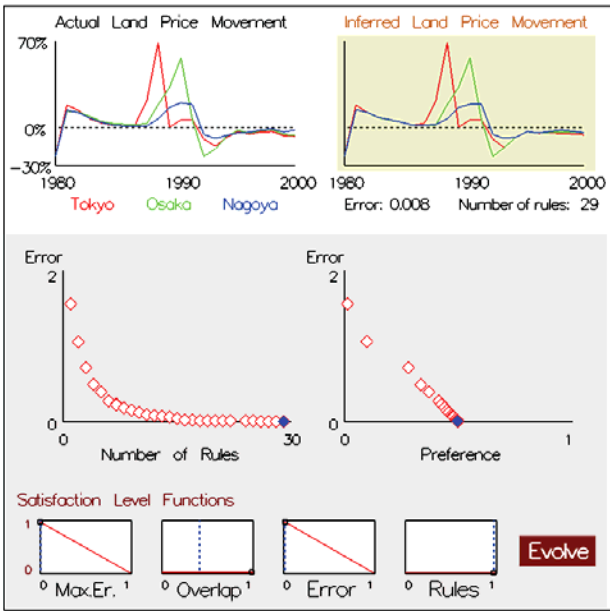

Figure 8: Example 3.

4 Conclusions

In this paper, we incorporated user's preference into multiobjective fuzzy modeling. We proposed a preference function composed of four satisfaction level functions. We utilized this preference function as an additional objective in an EMO algorithm. Through a case study, we demonstrated that a user can interactively specify satisfaction level functions during the evolution. We also showed that the user can obtain an accurate and interpretable fuzzy rule-based system based on his/her own preference.

In our case study, we intuitively selected four criteria to represent user's preference. Further studies are needed to choose appropriate criteria. We also have other interesting research issues to be discussed in future studies such as the visualization of multi-dimensional data and the minimization of human user's fatigue caused by the interaction with our system. The latter includes automated preference modeling.

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