

**Editor-in-Chief** 

### Mathware & Soft Computing

### The magazine of the European Society for Fuzzy Logic and Technology

### **Hot Topics**

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### Message from the Editor-in-Chief (December 2014)





Here it is the next issue of our Mathware&Soft Computing online magazine, the last one of this year which is about to end. And, as usual, we have tried to make it as interesting as possible so that you can find some time these days to enjoy its reading and, why not?, maybe preparing some collaboration for it.

As usual, we open the new issue with an interview among two very relevant figures of our communities. Lluís Godo and Francesc Esteva share with us their opinions and thoughts from the Artificial Research Center in Barcelona. And in order to know better this place where excellent scientists have carried their work, Ramón López de Mántaras proposes us his view, which I am sure all of us will find of great interest.

Last July the IPMU conference took place in Montpellier. At that time, the EUSFLAT Assembly was held and, in this issue, we include the minutes so that every reader can have first-hand knowledge of them. Moreover and also closely related to the EUSFLAT activity, we present a report on the relaunching of the EUSFLAT working group on image process-

ing, prepared by Irina Perfilieva, who is working very hard, with some other colleagues, to make this possible.

In this issue we cross the ocean to bring a report about one of the most significant journals for our community. Chin-Teng Lin writes for all of us a short document on IEEE Transactions on Fuzzy Systems, of which he is editor-in-chief.

But, of course, this is the EUSFLAT community magazine. For this reason, we follow trying to publicize the work of our researchers and, specially, the youngest ones. For this reason, we include in this issue abstracts of some of the works which we presented and supported by EUSFLAT grants. We hope that, in this way, the very high-quality research which is being developed by our students becomes widely known, maybe offering the possibility of future collaborations, opening new research lines, etc.

On the other hand, this issue also includes an emotive note, with a tribute, by Bernadette Bouchon-Mennier, to our sadly passed-away colleague Elie Sanchez, to whom many of us has the real pleasure to meet.

And of course, conference reports, books announcements, calls for papers...all those many activities in our full of life society. Activities that we hope to be even better reflected in forthcoming issues by your collaboration, your contributions and your ideas. So, many thanks to all those who have made this new issue possible and, for all the others, just recall that this is also your magazine and its pages are open to whatever you may consider of interest for our community! So here it is this new issue. Enjoy it!

Humberto Bustince Editor-in-chief

### Message from the President (December 2014)



Dear EUSFLAT members,

I open this letter with the memory of an esteemed colleague who passed away recently, in March 2014. Elie Sanchez was a Professor at the Université Aix-Marseille II, and one of the earliest contributors to the scientific community of fuzzy sets. After achieving a PhD in Mathematics, in 1972, he got a PhD in Human Biology in 1974, both of them from the Université Aix-Marseille. The latter PhD thesis had the title: "Equations de relations floues" (Fuzzy relational equations), with the first proposal of application of fuzzy sets in the medicine field, related to medical diagnosis assistance.

Elie Sanchez has been a Visiting Research Associate at the University of California at Berkeley in 1976-1977 and 1981-1982, where he started his collaboration with Lotfi A. Zadeh, with whom he co-edited a book on "Approximate reasoning in intelligent systems, decision and control" published by Pergamon Press in 1987. Later, in 1997 he co-edited a second book, titled "Genetic algorithms and fuzzy logic systems" published by World Scientific.

In 1978 he organized in Marseille one of the very first conferences dedicated to fuzzy sets, the International Colloquium on the Theory and Applications of Fuzzy Sets. He also chaired the IFAC Symposium on Fuzzy Information, Knowledge Representation, and Decision Analysis in Marseille, in1983. In 1989 he started to contribute at the development of fuzzy control, also collaborating with companies such as PSA in France and NASA Ames Research Center in USA. He founded the Neural & Fuzzy Systems Institute in Marseille, aimed at both managing industrial collaborations and developing theory.

From 1989 to 1991 he has been the President of the International Fuzzy Systems Association, and in 2001 he has been awarded of an IFSA fellowship. He has also received the Grigore MOISIL Gold Medal and Award in 1995. His more recent research interests were related to Fuzzy logic in the Semantic Web and to Web Intelligence.

In addition to his outstanding professional achievements, I would also like to remind the valuable human qualities of

#### **GABRIELLA PASI**

Elie Sanchez; we will always remember him fondly.

This year is coming to an end, and I would like to remind to all members that in 2015 two events very important for EUSFLAT will take place. The first will be the joint 16th World Congress of the International Fuzzy Systems Association (IFSA) and 9th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT) that will take place in Gijón, Asturias, Spain (June 30th - July 3rd, 2015): http://www.softcomputing.es/ ifsa-eusflat2015/. The deadline for papers submission is January 16th, 2015. I remind you that the EUSFLAT conference supports: some student grants (rules for applicants can be found at: http://www.eusflat.org/ members\_grants.php), the Best Student Paper award, and the EUSFLAT Best Ph.D. Thesis award (rules for applicants can be found at: http://www.eusflat.org/awards.php). The second event that we plan to organise in September 2015 will be the first Summer School on Fuzzy Set Theory and Applications. We will send you soon more information about this event.

In relation to the scientific activities promoted by EUSFLAT I would like to outline the key role played by the EUSFLAT Working Groups; their activities are extremely important to support research and collaborations within our scientific community. For this reason I kindly invite you to check the list of Working Groups on the EUSFLAT Website and to join those within which you could contribute (http://www.eusflat.org/research\_workinggroups.php).

EUSFLAT in 2014 has been composed of 252 members, and it keeps maintaining a scientific network with 8 scientific societies and 31 conferences that are directly linked to EUSFLAT. For all the above reasons I kindly ask you to help us to invite your colleagues to join the EUSFLAT project!

I would like to close this letter by thanking all colleagues and friends who have supported and who keep supporting the activities of the Society: in particular, Humberto Bustince and his team who actively work to maintain our magazine updated and appealing, Jorge Casillas who takes care of the EUSFLAT Web Site, and, last but not least, Edurne Barrenechea who is so available, precise and effective in her important role of EUSFLAT treasurer. A big thank to all the other colleagues and friends of the EUSFLAT Board, as well as to all EUSFLAT members for making the Society alive!

I close this letter by wishing to all the EUSFLAT members and their families a Merry Christmas and a Happy and Healthy new year, where our Society can grow and offer more opportunities to collaborate.

Gabriella Pasi President of EUSFLAT

### **INTERVIEW**

### Dialogue between Francesc Esteva and Lluís Godo



Francesc Esteva and Lluís Godo

How did you start doing research and how did you got into fuzzy sets and fuzzy logic?

**E** My landing in the field of fuzzy sets was very smooth. I started my research activity when I took PhD courses of Prof. Francesc de Asis Sales, professor of Probability and Statistics at the University of Barcelona, and lover of logic, something very rare in a faculty of Mathematics in the mid 70s in Spain. In that PhD course, Prof. Sales provided different algebraic results related to non classical connectives, in particular, related to the intuitionistic negation. I liked the topic and I wrote my PhD thesis on negations in complete lattices, defended in September 1974. The thesis was mainly of an algebraic nature, but with some logical motivations. By that time, I was with the Mathematics department of the Architecture School at UPC (Barcelona), whose director was Enric Trillas, who began working on fuzzy sets and stablishing strong relationships with pioneers of Fuzzy Sets researchers, specially with Lotfi Zadeh. I started then to discuss with him about fuzzy sets, and when he published his paper on negations functions I realised that some results of my thesis could be applied to fuzzy sets as well. Then I translated and expanded the results in two papers with Xavier Domingo and Enric Trillas, and we presented a communication on this topic at ISMVL-1981 in Oklahoma, USA (the first international conference related to fuzzy sets I attended). After that, we actively participated in different local events (Catalan conferences on logic, department seminars, FISAL workshops in Mallorca). It was a very fruitful period, with plenty of events and full of hope in all senses. The country was looking for new horizons, and research was gradually making its own way. And we felt like pioneers, and in a sense I think we were so since we opened a very important path for those who came behind.

L. Some years later, in 1983, and thanks to Francesc, I joined Enric's department to replace a colleague that was in a sabbatical year (my first job!). In that department, I found

a very pleasant atmosphere and very active in research, especially on fuzzy sets. I remember the first paper related to fuzzy sets I read was given to me by Enric, it was Zadeh's paper on probability of fuzzy events. After, I went through all Zadeh's papers and started my own research on approximate reasoning, under the supervision of Llorenç Valverde, that eventually led some years later to my PhD, defended in 1990.

### What research topics you were first interested in?

F. Probably I would have liked to explore fuzzy engineering aspects, but the truth is that I am very oriented towards formal subjects like math and logic, indeed I have always worked on formal aspects of fuzzy logic through several stages. I began with the study of the operations on [0,1] that were in fact related to logical connectives of manyvalued logics underlying fuzzy logic. This period corresponds to works on negations already mentioned, and on algebraic structures for fuzzy sets. I remember as a nightmare the time when a group of researchers of the department were interested in studying fuzzy sets as a topos. I believe my allergy to category theory comes from that experience. Then, my logico-algebraic interest took me to study Okham algebras (distributive lattices with a negation fulfilling the De Morgan's laws that are related to the implication-less fragment of fuzzy logics). This was the topic of the PhD thesis of Pere Garcia, about which I remember a pleasant conversation with Helena Rasiowa during ISMVL-1988 in Mallorca.

L. In those early years, I was mainly interested in studying different patterns of approximate reasoning, using many-valued logics connective arising in residuated structures on [0, 1], as well as their formulation in terms of fuzzy truth-values. From the distance, that kind of research may appear now as not very systematic, rather as done in a case by case basis without clear formal grounds, but I think this was the price one had to pay (and I do not regret) to have somehow naively started exploring a relatively new field.



A moment at FUZZ-IEEE 92 in San Francisco

And then, the CEAB (Center for Advanced Studies of Blanes of the CSIC) was created in 1985, together with a research group on Artificial Intelligence led by Ramon Lopez de Mantaras, Jaume August, Josep Aguilar and Settimo Termini, and you both ended up there for different reasons. How did this influenced your research?

**E** I joined the CEAB from the very start as a part-time researcher and participated in the first CEAB research project, led by Josep Aguilar, on expert systems and management of vagueness and uncertainty. In 1987 I fully joined the CEAB and became its director. When I arrived the research of the Artificial Intelligence group was devoted to the development of the expert system MILORD that Ramon López de Mantaras and Carles Sierra had designed and whose inference engine was based on fuzzy logic.

L. I also joined the CEAB very early, in 1987, and I immediately started to collaborate with C. Sierra and R. Lopez de Mantaras on the logical aspects of the expert system MILORD, that was being applied in a medical application to diagnose Pneumonia (with A. Verdaguer as medical expert). It quickly became an ideal platform to advance in many practical but also theoretical aspects of fuzzy logic. For instance, we realised that defining combinations of fuzzy linguistic terms for inference purposes was in fact equivalent to study abstract operations on finite sets of truth values. In this way, it began our interest on t-norms, s-conorms and negations on finite scales. That was a very exciting period, attending my first international conferences, and making contacts with renown specialists. It was in this time that I initiated my scientifical and personal relation with Francesc, discussing new research topics related to developments of MILORD system.



Celebrating Zadeh's Dr. Honoris Causa in Oviedo

E Indeed, with the idea of better managing large knowledge bases, the MILORD system evolved from a flat to a modular system, and we proposed each module in a knowledge base could have associated a possibly different many-valued logic (varying the number of truth-values and operations) and we studied how these logics could be combined in a consistent way. This lead us to Goguen and Burstall's theory of institutions (a semantic framework to define very general logical systems with the idea of bringing logic closer to computational) and we showed that our modular systems of many-valued (fuzzy) logics perfectly fit this framework. On the other hand, our joint interest in approximate reasoning models led us to follow very closely the work by Enrique Ruspini based on semantics for fuzzy sets based on

fuzzy similarity relations. His ideas greatly influenced our joint developments with Didier Dubois and Henri Prade on similarity-based logics (and much later they also led to Ricardo Rodríguez's thesis on similarity-based reasoning) and their application to case-based reasoning and fuzzy interpolation. More recently, we have also worked on this subject with Bernadette Bouchon and her group at Paris and with Irina Perfilieva.



Francesc Esteva, Petr Hájek and Lluís Godo

L. I would also like to mention that in this period my participation in the European projects DRUMS (Defeasible Reasoning and Uncertainty Management Systems) and DRUMS-II had a significant influence in my research. Philippe Smets, leader of these projects managed to gather most of outstanding European researchers on knowledge representation and uncertainty reasoning (e.g. Dubois, Prade, Lang, Gabbay, Hunter, Clark, Wilson, Sandewal, Doherty, Kruse, Moral, Nebel, Treur, Tan, Meyer, and many others). That was an amazing opportunity for learning a lot from the interaction with all of them and for very interesting joint collaborations, as well as an endless source of inspiration, that lasts until today. For instance, our long-lasting close and fruitful relationship to Didier Dubois and Henri Prade comes from those days.

And in the early 90's you got into contact with Petr Hájek, from Prague. What do you remember about those early visits to Prague and receiving visitors from there at the CEAB?

L. Knowing Petr Hájek has been decisive in our lives as 'fuzzy logicians', we have actually been very fortunate to learn a lot with him (never enough!) and to be witnesses of the birth and development of fuzzy logic in a formal sense as a new discipline, what nowadays is known as mathematical fuzzy logic. As it is usually said, we were very lucky to be in the right place in the right time. Indeed, thanks to the first contact we had with Petr Hájek (thanks to the late prof. Jiri Becvas) was in a short visit to Prague of Carles Sierra and myself in the summer of 1987 while we were in research stay in Munich, and the main motivation was to talk about uncertainty models for expert systems. After that first contact, soon after the 'velvet revolution' in Prague, the Czech Academy of Sciences and the CSIC signed a joint bilateral agreement for exchanging visits, and in 1991 that we could make a first visit to Petr in Prague. I remember that by that time he was at the Mathematical Institute, and Petr's office was in reconstruction and we could only have short but intensive discussions in our student residence there. As far as we know, it was in that period that Petr Hájek got really interested in fuzzy logic. In this way, we could start a long, illuminating and continuous cooperation (for almost twenty years) with Petr Hájek and his group at the Institute of Computer Science in Prague.



Roberto Cignoli, Francesc Esteva and Lluís Godo

E I will always remember the impact that Petr Hájek made on me because of his appearance of a venerable bearded man, his smile that made one feel comfortable with him, his deep scientific ideas and his open mind. He was very interested in discovering which "logic" was behind Zadeh's fuzzy logic, so to be able to provide fuzzy sets and fuzzy logic with well-founded grounds. With him we revisited everything we had done, from a new and deeper logical perspective. We worked on a modal approach to possibilistic logic, we modeled reasoning about uncertainty by theories in rational Pavelka logic, and we helped Petr axiomatizing Product logic, the third of the fuzzy logics based on the three most prominent continuous t-norms (Lukasiewicz, Gödel and Product t-norms). Once Lukasiewicz, Gödel and Product logics were defined, Hájek introduced the Basic Fuzzy logic (BL) in his celebrated 1998 book as the logic of continuous t-norms, for which we, together with Roberto Cignoli and Antoni Torrens, proved completeness with respect to the intended semantics, that is, BL theorems coincide with the common tautologies for all continuous t-norms and their residua. As Hájek says in his book, BL is the kernel of fuzzy logic, although Zadeh's agenda goes far beyond the typical agenda of a particular many-valued logic. It is with this aim that Hájek devotes the second half part of his book to the analysis of concepts and topics in Zadeh's agenda (like as fuzzy if-then rules, linguistic modifiers, linguistic quantifiers or uncertainty modalities) that are not directly covered by the usual treatises on many-valued logic.

And shortly after Petr Hájek's BL, you introduced the logics MTL and L , extensions of fuzzy logics with an involution, with truth-constants, etc. How did they arise and what do they bring about?

**E** Actually, a keystone in the emerging of mathematical fuzzy logic was in the late 90s the COST Action "Many-Valued Logics for Computer Science Applications" (from 1997 to 1999), led by Luisa Iturrioz, was indeed a unique and greatly beneficial occasion to put into contact most scholars across Europe interested in many-valued and fuzzy logics. It was in the frame of this project that the MTL logic came out. The main idea was the observation that if one weakens the axiom

of divisibility in BL (that forces the continuity of the strong conjunction on [0,1]), most of the properties are preserved, especially the fact that the logic is complete with respect to the class of linearly ordered algebras of the corresponding variety. Therefore the resulting logic seemed appropriate as the logic of left-continuous t-norms and their residua, as they it was later proved by Sandor Jenei and Franco Montagna. Actually, all this has its roots in Ulrich Höhle's excellent work on commutative residuated monoids, over which he defined the so-called monoidal logic, as a suitable logical system underlying fuzzy sets. However, if we take the real unit interval [0, 1] as the proper range of membership degrees for fuzzy sets, and hence of truth-values for the logic, the underlying logic turns out to be MTL (a logic strictly contained in the Monoidal logic), which takes into account the linear order of the truth-values. The name MTL (for Monoidal tnorm based logic) was decided in an informal discussion with Siegfried Gottwald, Petr Hájek, Lluís and myself during the final meeting of the above mentioned COST Action in Vienna in 1999. The idea was to keep the term "monoidal" from Höhle's monoidal logic and to add the term "t-norm based" to refer to the intended semantics given by left-continuous t-norms. This logic and some of its extensions was the main subject of the PhD thesis of Carles Noguera, jointly supervised with Joan Gispert. Also, the implication-less fragments of MTL were the main research topic of the PhD thesis of Angel Garcia-Cerdaña, jointly supervised with Ventura Verdú.



Lluís Godo, Franco Montagna and Francesc Esteva

Thanks to a conversation with Ewa Orlowska, we got to know the works of Hiroakira Ono about residuated logics. We already knew about Hiroakira Ono's papers on logics without contraction but not about his last survey on residuated lattices. And we realised we were working on essentially the same hierarchy of residuated logics. We wrote a paper comparing both hierarchies and making clear what were the coincidences (with different names) and the differences. I

think that work helped in clarifying a landscape of a family of logics and their relationships, and to properly locate Hájek's and our own ones' contributions in the landscape of substructural logics. Nowadays, there is a lot of activity in this field, where many logics are interesting by themselves and some may be also interesting for their relation to fuzzy logic. In any case, we think mathematical fuzzy logic has today its own place in the field of "logic". Today we can present our works without problems in fuzzy logic venues but also in traditional logic forums, and this seems to us the paradigm of what everyone working in the fuzzy field should do, to have one foot in both the fuzzy community and in the corresponding application field.

L. The beginning of the 2000s was a very active period, with continuous new advances in the field, besides Hájek's book, new monographs by Vilém Novák et al., Siegfried Gottwald and Giangiacomo Gerla appeared. We also defined a number of new logical systems, for instance the ones resulting from the addition of an involutive negation to logics whose residual negation is not involutive, and the logic L resulting from gathering both the connectives Lukasiewicz and Product logics altogether. In both cases the motivation came from what we knew about fuzzy logic. In the first case one can define a dual s-conorm from a given involutive negation. In the other case, the logic L allowed to have in a same logic connectives related to the addition and subtraction (Lukasiewicz connectives) and connectives related to product and division (Product connectives), thus a logic with a very high expressive power. It is worth to recall that the paper on the L logic was a joint work with Franco Montagna, and this was in fact due to Petr Hájek. Indeed, Petr was aware of a preliminary works of ours and of another by Franco as well on the same topic, and he put us in contact with Franco. We quickly agreed on how to proceed since our preliminary works were rather complementary, and we published our first joint paper shortly after. This first contact with Franco (only by email) was later followed by a very friendly and fruitful collaboration since then, for instance papers on standard completeness of several extensions of MTL, postive fragments of fuzzy logics, or the axiomatization of the logics defined by any continuous t-norm (hence any ordinal sum the three basic components) are some of the results of that collaboration.



Participants of COST 1999

And what have you been doing more recently?

**E** In the last years we have continued trying to model from a logical point of view different issues related to fuzzy reasoning. An interesting work (in collaboration with Carles Noguera) has been axiomatizing truth-hedges (modeling linguistic modifiers), following the early works of Peter Hájek and Vilém Vichodyl, and providing a general axiomatization. Together with Félix Bou and Ricardo Rodríguez, we have also been working for several years on modal extensions of many-valued logics, a rather unexplored domain by the way, are currently working right now in modal extensions of Gödel and Product logics.



Francesc Esteva, Lofti A. Zadeh and Daniele Mundici

L. At this point, in relation to fuzzy modal logics, let me recall a nice story. We were really amazed when Lotfi Zadeh contacted us in December 2012 since he had read some paper of ours in this topic and had some questions on how these logics were related to Possibility theory. Then an extremely interesting and long exchange of questions, answers, arguments and opinions by email began among us that lasted for more than one year, where Lotfi's smart questions helped a lot to clarify ourselves several aspects of the formalism. Eventually, Lotfi published a short paper entitled "A note on modal logic and possibility theory" in Information Sciences. We will be always honoured and indebted to Lotfi for dedicating that paper to us.

**E** We have also studied some other logical systems in order to better explote many-valuedness since the usual notion of consequence in fuzzy logics do not take full advantage of being many-valued, as they focus on the truth value 1 (the truth) and not on other intermediate truth values. A first approach to circumvent this, while keeping the truth-preserving framework, is to introduce truth constants into the language. This methodology goes back to Pavelka, further developed by Hájek and Novák basically over Lukasiewicz logic. We have extended this approach to a more setting and studied the corresponding logics. A second approach we have been working on more recently is an alternative notion of logical consequence that preserves degrees of truth. The basic idea is that in a many-valued logic setting, one could require for a deduction to hold that the truth value of the conclusion has to always be greater than or equal to that of the premises. This gives rise to a stronger notion of logical consequence that has been called 'logic that preserve degrees of truth'. We have studied these alternative fuzzy logics with colleagues from the logic department of the University of Barcelona.

L. Actually, the logic preserving degrees of truth are a key component in a current European IRSES project with members of the logic group at the University of Campinas (Marcelo Coniglio, Rodolfo Ertola), where we study the relationship between fuzzy and paraconsistent logics. It is clear that usual fuzzy logics (that preserve the full truth) are not paraconsistent but it turns out that the ones preserving degrees of truth can be paraconsistent if they are not pseudo-complemented. We are currently studying, for Lukasiewicz semantics, intermediate logics between the 1-preserving logic and the one that preserves degrees of truth. We have seen that there is an infinite number of intermediate logics, some of them paraconsistent, some of them explosive.

### You have worked together for many years, but have you also worked on different topics?

F. Since the early 90s when we began to work together, I think only a few articles of mine, resulting from the theses of Pere Garcia, Carles Noguera and Angel Garcia-Cerdaña, are not co-authored by Lluís. In recent times, however, there is an issue that I have not worked with Lluís, fuzzy description logics, but with Felix Bou, Marco Cerami and Angel Garcia-Cerdaña. In this topic, following the seminal work of Petr Hájek, we have taken advantage of the already available on mathematical fuzzy logic to define fuzzy description logics (FDLs) in parallel to the classical case. We have several articles defining the language, providing results on decidability and complexity, and defining a translation between FDLs and modal many-valued logics. We think this is a very interesting line to follow. Consider for instance the results found showing that the complexity of classical description languages is the same as in those based on finitely-valued logics. Therefore, it seems it could be possible to convince researchers on classical DLs to use a finite number of truth-values instead of only the classical true and false, and thus increasing their representation power. Unfortunately, the severe budget cuts for research in the last years in our country (basic research will need many years to recover!) has left us orphans of collaborators and, doctoral students. In any case, there are other groups working along this research line and we are happy for the fact that this can represent an increase on the use of fuzzy techniques in knowledge representation languages.

L. In the last ten years or so, I have done some independent work mainly along two research lines: computational argumentation and on uncertainty logics to reasin with fuzzy events. Together with Teresa Alsinet (a former PhD student of mine) and colleagues from the Unviversity of Lleida we have also worked to some extent on computational argumentation, mainly around Guillermo Simari's DeLP argumentation system, allowing arguments to be tainted with vagueness and possibilistic uncertainty in order to model their strength. On the other hand, with Enrico Marchioni and Tommaso Flaminio we have been developing a fuzzy logical approach to reason under uncertainty on many-valued events, extending initial ideas in joint papers with Petr Hájek on introducing fuzzy epistemic modalities to model different kinds of beliefs (probability, possibility, Dempster-Shafer belief functions). This kind of approach has taken also advantage of deep results by Daniele Mundici, Franco Montagna and colleagues on the generalization of finitely-additive measures on MV-algebras (the algebras related to Lukasiewicz logic), also known as states, and on the generalization of de Finetti's betting metaphor to the many-valued framework.

Some final thoughts on the subject? from your perspective, how do you envisage the research on fuzzy logic in the next future?

E In my opinion, and I think Lluis shares this idea, researchers in the field of fuzzy sets and fuzzy logic should have a foot in both the fuzzy world and in the corresponding application field, otherwise they can end up making a sort of isolated research. This is a necessary condition to get recognition from the non-fuzzy world. To some extent, I think the community of formal or mathematical fuzzy logic have managed to do this. On the other hand, I have also mentioned the fact that Zadeh's agenda for fuzzy logic is much broader than that of a purely many-valued logic. There are many concepts on this agenda that many-valued logic does not deem as important. From my point of view one of them that until now we have not been able to address is the issue of how to deal with fuzzy quantifiers within mathematical fuzzy logic (MFL). For instance, it is somewhat surprising that MFL only have considered the generalization of classical universal and existential quantifiers, the ones defined using infima and suprema of truth-values. But now there are a good background and well founded studies over which I believe one can start studying new elements and concepts in Zadeh's fuzzy agenda.

L. From my side, only two final remarks. The first one is that we feel really very fortunate to have met and collaborated along our research career with first class researchers and better persons, like Roberto Cignoli, Antonio Di Nola, Didier Dubois, Petr Hájek, Franco Montagna and Henri Prade, to mention only a few. The second remark is that I am very optimistic since that there is a very strong new generation of 'fuzzy' logicians, among them e.g. Stefano Aguzzoli, Libor Behounek, Félix Bou, Agata Ciabattoni, Petr Cintula, Tommaso Flaminio, Brunella Gerla, Enrico Marchioni, George Metcalfe, Carles Noguera, etc., that guarantee a bright future for the field of fuzzy logic.



Participants of ESTYLF 98



#### Francesc Esteva

obtained his master in Mathematics in 1969 and his PhD also in Mathematics in 1974. He dedicated some years to Mathematical education and he has been director of a journal devoted to this topic titled

L'Escaire. At the same time he was professor at the Architecture School of Barcelona. He was member of its Mathematical department where Enric Trillas began the research in Fuzzy Sets and Fuzzy Logic in Spain and he participate from the beginning in this research. His first paper in an International conference was a paper in the ISMVL'81 in Oklahoma city and from then he has published more than a 150 papers in journals, conferences and workshops mainly devoted to Fuzzy logic. During the nineties he participated actively in the group of fuzzy researchers and logicians lead by Petr Hájek that have created the Mathematical Fuzzy logic. From then he works mainly in this topic but he also have worked in similarity-based reasoning and its application to Case-based reasoning, in interpolative reasoning, in modal many-valued logic and in Fuzzy Description Logic. From the organizational point of view, in the eighties he was deputy director of the Architecture School of the Technical University of Barcelona. In 1987, he moved to the Spanish National Research Council (CSIC) as director of the Center for Advanced Studies in Blanes (CEAB), and in 1994 the Artificial Intelligence Research Institute (IIIA-CSIC) was created and he became his director until 2007. Moreover, in 1991 he participated in the creation of the Spanish Association for Fuzzy Logic and Technology (FLAT), and in 1995 he was elected President of this association. In 1997 he was elected as the first President of the European Association for Fuzzy logic and Technology (EUSFLAT). Now he is honorary member of EUSFLAT and, as retired researcher, he is Professor "at honorem" at the IIIA-CSIC.



#### Lluís Godo

is a Research Professor at IIIA-CSIC, the Artificial Intelligence Research Institute (IIIA) of the Spanish National Research Council (CSIC), Barcelona, Spain. He obtained his MSc degree in Mathematics from

the University of Barcelona (1979) and the PhD in Mathematics from the Technical University of Catalunya (1990). His main research interests include logics for Artificial Intelligence (AI), in particular graded uncertainty reasoning formalisms, mathematical fuzzy logic, and argumentation systems. He is author of more than 150 publications in international journals and conferences. He has been Program cochair of Fuzz-IEEE'97, chair of the 8th European Conference on Qualitative and Quantitative Reasoning under Uncertainty ECSQARU 2005, and program co-chair of the 3rd Intl. Conf. on Scalable Uncertainty Management SUM 2009. He has served in the PC of numerous national and international AI-oriented conferences (ECAI, ECSQARU, IJCAI, KR, UAI, AAMAS, CCIA, CAEPIA) and more fuzzy logic-oriented conferences (EUSFLAT, FUZZ-IEEE, IFSA, IPMU, LATD, ESTYLF). He has been guest editor of several special issues of international journals and volumes, and he is currently Area Editor of the journal Fuzzy Sets and Systems (Elsevier), Associate Editor of Soft Computing (Springer) and member of the editorial board of the Artificial Intelligence Journal (Elsevier). He is an ECCAI Fellow and an IFSA Fellow. He is past vice-president of the European Society for Fuzzy Logic Technologies (EUSFLAT) and of the Catalan AI association (ACIA).

### **ACTIVITY REPORT**

# Fuzzy logic / fuzzy systems research and applications at the Artificial Intelligence Research Institute (IIIA)

### Ramon Lopez de Mantaras

The Fuzzy Logic/Fuzzy Systems activities at the IIIA started in 1985 when Professor Enric Trillas, then President of the Spanish National Research Council (CSIC), asked Prof. Ramon Lopez de Mantaras to found an AI department at the newly established Centre of Advanced Studies located in Blanes. With the collaboration of Dr. Jaume Agustí, from the Autonomous University of Barcelona, Prof. Josep Aguilar-Martin, from the CNRS, and Professor Settimo Termini, from the CNR, the AI research activities started at this centre. The group grew fast and in 1994 we became the Artificial Intelligence Research Institute (IIIA) and moved to a new building located in the campus of the Autonomous University of Barcelona.

At present, our research activities are structured around three departments: Learning Systems, Multi-agent Systems, and Logic, Reasoning & Search. However, in what follows we will focus on the activities related to the area of Fuzzy logic and Fuzzy systems, area in which Prof. Enric Trillas was the pioneer in Spain. We have structured these activities in three periods: The first 10+ years from 1985 to 1995, then the next 5+ years, from 1996 to 2001 and finally from 2002 till now. For additional information regarding all our contributions to AI we refer the reader to the "History" section of our website: www.iiia.csic.es/en/about\_iiia/history, as well as to the very recent paper "A Survey of Artificial Intelligence Research at IIIA" published in AI Magazine 35(3), Fall 2014.

### THE FIRST YEARS: 1985 TO 1995

### **Knowledge Based Systems**

The research on Knowledge Based Systems has been one of the initial interests of the group that has had continuity till today. Motivated by several real applications, we created, formalized and implemented languages to better represent uncertainty and imprecision, based on fuzzy and multivalued logics. These languages have been integrated in a two-generation tool (MILORD and MILORD II) on top of which most of the applications to real domains have been built.

MILORD was an expert system building tool developed between 1985 and 1989 within the framework of the PhD thesis of Carles Sierra. It allowed to perform different calculi of uncertainty on an expert defined set of linguistic terms expressing truth degrees. Each calculus corresponded to specific conjunction, disjunction and implication operators. The internal representation of each linguistic truth value was a fuzzy subset of the interval [0,1]. The different calculi of uncertainty applied to the set of linguistic terms, resulted in a fuzzy subset that was approximated to a linguistic truth value belonging to the set of linguistic terms. This linguistic approximation kept the calculus closed. This had the advan-

tage that, once the linguistic truth values had been defined, the system computed, the conjunction, disjunction and implication operations for all the pairs of linguistic truth values in the term set off-line, and stored the results in matrices. Therefore, when MILORD was run, the propagation and combination of uncertainty was performed by simply accessing these pre-computed matrices. This tool also used a meta-level language to represent the strategies of execution of modules containing domain rules. This meta-control language was the inspiration of some work done in Case-Based reasoning as well. MILORD has been used in the development of several real applications. The first application was the expert system PNEUMON-IA, on the diagnosis of community-acquired pneumoniae. This problem required extensive research in the area of uncertainty, to satisfactorily represent the lack of precise diagnostic procedures of the domain. It took two years to complete it. In 1988 it was validated, and the results presented in 1989 in an M.D. thesis by Albert Verdaguer. In 1987 we started another application in the area of rheumatological diseases and colagenosis. The more heterogeneous nature of the set of diseases included in this application forced us to develop more complex and declarative control structures to represent the dynamics of the reasoning that the expert needed to model the diagnostic processes. The application was validated in 1989 and the results published in the M.D. thesis of Miguel Belmonte.

We had also been involved in applications to industrial problems. In particular, from 1988 to 1992, we developed, based on MILORD, a successful diagnostic system for defects in TV screens manufactured by PHILIPS. This research was done in the framework of two projects (IPCES-I and IPCES-II) funded by the ESPRIT I and ESPRIT II European research programs. IPCES-I was one of the very first European projects funded in Spain. The diagnostic system was connected to a vision system capable of detecting different categories of defects, and to an information system that provided data from the process plant. Using this combination of information a ranking of the most plausible causes of the defect was generated as output. The work around the MILORD expert systems building tool had a very significant international impact and was awarded the Digital Equipment European Artificial Intelligence Research Award.

MILORD was improved and extended, becoming MILORD II, an architecture for Knowledge Based Systems that combined reflection and modularization techniques, together with an approximate reasoning component, to be able to define complex reasoning patterns at large. Its development started in 1989 and constituted the main component of Josep Puyol's Ph. D. thesis. A Knowledge Base in MILORD II

consisted of a set of hierarchically interconnected modules. Each module contained an Object Level Theory and a Meta-Level Theory interacting through a reflective mechanism. From the logical point of view, MILORD II made use of both many-valued logic and epistemic meta-predicates to express the truth status of propositions.

An application that was developed and validated using MILORD II was Spong-IA, an automatic classification tool for marine sponges. It covered all the atlanto-mediterranean taxonomy up to the level of family and a part of it up to the level of species. It passed an international experts validation process with great success. The main results of this work were presented in a Ph. D. in Biology by Marta Domingo.

### **Fuzzy and Multiple-valued Logics**

According to Zadeh, the Fuzzy logic term is used, at least, with two different meanings. Fuzzy Logic in broad sense refers to methodologies involving Fuzzy Sets and Possibility theories, whereas in narrow sense it refers to the various formal logical calculi underlying Fuzzy Set Theory. The theoretical research done in our group on Fuzzy Logic has covered both aspects. The main contributions have been in the following two subjects:

Fuzzy truth values

The work on modeling inference in Fuzzy Logic using the Fuzzy Truth Values formalism started at the IIIA in 1990 with the PhD thesis of Lluis Godo. This formalism allowed the implementation of some inference patterns without the need to specify particular possibility distributions to represent the fuzzy statements involved in such inference patterns. It was shown that Fuzzy Truth Values play the same role that classical truth-values do in classical or many-valued logic. In this direction, we also studied the closure system of inference operators in the above formalism as well as a semantic formalization of fuzzy logic as logic with fuzzy truth-values. *Multiple-valued Logic* 

The investigation of different fuzzy (or many-valued) logics, in the narrow sense, was motivated by a fruitful cooperation since 1993 with the Institute of Computer Science of the Czech Academy of Sciences, led by Prof. Petr Hájek. This collaboration resulted in a number of significant publications about different systems of many-valued logic and their relation to main uncertainty calculi, such as probability theory or possibilistic logic.

### **Similarity Logic**

Similarity relations, as generalizations of equivalence relations, were defined by Zadeh in the late sixties. Most of the early work dealt with the application of these relations to cluster analysis. In the eighties Enric Trillas introduced a generalization of Zadeh's definition and Trillas and Valverde related similarity relations to equivalence connectives in fuzzy logic. In the nineties this type of fuzzy relations started to be used in order to obtain a semantics for fuzzy logic and to build a logical setting for dealing with sentences like "close to p", "not far from p" or "similar to p" being p a proposition. In both issues the contribution of our research group was very relevant.

In the early 90s Ruspini published his studies on a semantics for fuzzy logic based on similarity relations. Based on this, Esteva, Godo and García proposed a definition of a similarity logic as a propositional logic based on similarity relations. A complete analysis of the relations between this

logic and the fragment of necessity-valued possibilistic logic and fuzzy-truth-valued logic was also achieved.



On the other hand, the concept of similarity was also used by researchers of the Institute, in cooperation with Dubois and Prade, to define graded consequence relations corresponding to different levels of approximation. The main idea underlying this approach was to approximate every classical proposition p by a fuzzy set of interpretations in such a way that the alpha-cuts of this fuzzy set provide a set of approximations of p. As expected, approximation in degree 1 coincides with p and approximation in degree 0 coincides with the classical set of all interpretations. In this setting, p entails q to the degree alpha if p classically entails the alphaapproximation of q. The results of the work done along this research line were both theoretical and practical. From the theoretical point of view, we studied the properties, a syntactical characterization, and a formalization, in a multi-modal and a multi-valued setting, of these graded entailment relations. From the practical point of view our results were also of interest. A framework for interpolative reasoning based on graded entailment was developed also in collaboration with Dubois and Prade, and applications to case-based reasoning, as well as to analogical reasoning, were developed. We were the first to incorporate fuzzy techniques within a Case-Based system in 1989.

### Fuzzy Logic for mapping unknown environments using autonomous mini-robots

An interesting application of Fuzzy Logic, undertaken in our Institute within the framework of the PhD work of Maite López-Sánchez, was to the problem of the acquisition of maps of unknown environments by means of a group of autonomous mini-robots. The problem of collective map generation is to obtain the most plausible position of walls and obstacles based on the perception of several mini-robots. The mini-robots detected portions of walls or obstacles with different degrees of precision depending on the length of the run and the number of turns they have done. The main problem was to decide whether several detected portions, represented by imprecise segments, were from the same wall or obstacle or not. If two segments were from the same wall or obstacle, a segment fusion procedure was applied to produce a single segment. This process of segment fusion was followed by a completion process in which hypothesis were made with respect to nonobserved regions. The completion process was achieved by means of hypothetical reasoning based on declarative heuristic knowledge about the orthogonal environments in which the mini-robots evolve. Finally, an alignment process also took place so that, for example, two walls separated by a doorway were properly aligned. All these operations were based on modeling the imprecise segments by means of fuzzy sets. More concretely, the position of the wall segment was a fuzzy number and the length a fuzzy interval. The main advantage of using fuzzy techniques was that the position and imprecision of the resulting fused segments could be very easily computed. Furthermore, using Fuzzy sets to model the imprecision about the position of obstacles was very appropriate.

### THE CONSOLIDATION: 1996-2001

### Foundations of mathematical fuzzy logic

Fuzzy logic until very recently lacked a formal basis. We have done significant research on fuzzy logic ńin the narrow senseż with remarkable results, due in part to the already mentioned fruitful collaborations with Prof. Petr Hájek and, more recently, with Prof. Montagna and Prof. Cignoli. The main results obtained concern the axiomatization of several t-norm based residuated logics: product logic, completeness of Hájek's basic fuzzy logic BL, residuated logics with involutive negation, Lukasiewicz Product logic L $\pi$  and Monoidal t-norm based logic MTL. Another important result has been the modeling of probability in the fuzzy logic setting and the expression of fuzzy inference as deduction in some of these types of logic.

### Similarity-based reasoning

The notion of similarity among knowledge states plays an important role in different inference patterns of approximate reasoning. Two relevant examples are the reasoning mechanisms used in fuzzy rule-based systems and in casebased reasoning. A fuzzy rule-based system interpolates rule consequents according to the degree of match between actual variable values and those in the rule premises. In doing so, the system extends the domain of application to system's states that are similar to those described in the fuzzy rule base. On the other hand, case-based reasoning techniques follow an analogy principle which states that similar problems have similar solutions, leading - naturally - to a formalization using similarity-based reasoning. Research on similaritybased reasoning, in close collaboration with the group of Profs. D. Dubois and H. Prade, has focused on two major issues:

#### Logical foundations of similarity-based reasoning

We have addressed several fundamental problems ranging from semantic to syntactic considerations, one being based on two graded similarity-based consequence relations, which allow an interpolation mechanism to be defined, and another on graded logics, both classical and many-valued, for which completeness results were obtained. Their relation to other types of graded logical formalism, like possibilistic logic, have also been considered.

Similarity-based reasoning and case-based reasoning and decision

We have used fuzzy set techniques, based on fuzzy similarity relations, to formalize some common problems which appear in case-based reasoning, such as retrieving the most relevant cases, or getting a more flexible adaptation of past solutions by interpolating them. A logical modeling of the inference patterns involved in case-based reasoning, using the

similarity-based consequence relation formalism, were also introduced. Regarding case-based decision theory, Gilboa and Schmeidler had proposed a new approach to decision theory based on similarity, rather than probability, where the utility function is defined on partially described situations in terms of their similarity with previously experienced decision. Using fuzzy similarity relations and possibility theory, a new qualitative decision model was proposed, closely related to Dubois-Prade's possibilistic decision theory, and with an axiomatic basis. Extensions to this latter model were also investigated.

### Case-Based Reasoning application to expressive music synthesis

One of the most successful and widely cited CBR system developed at our institute is an application to the synthesis of expressive music performances. The problem solving task of the system is to infer, via imitation, and using case-based reasoning, a set of expressive transformations to be applied to every note of an inexpressive musical phrase given as input. To achieve this, it uses a case memory containing human performances and background musical knowledge. The score, containing both melodic and harmonic information, is also given. The expressive transformations to be decided and applied by the system affect the following expressive parameters: dynamics, rubato, vibrato, articulation, and attack. The similarity reasoning capabilities provided by CBR allow the system to retrieve those notes in the case base of expressive examples (human performances) that are, musically speaking, similar to each current inexpressive note of the input. We developed a fuzzy approach to combine a set of solutions from several retrieved cases into a single solution to be applied to every note of the inexpressive inpout in order to render it expressive. The system is connected to software for sound analysis and synthesis based on spectral modeling as pre- and post-processor. This allows the obtained results to be listened to. These results clearly showed that a computer system can indeed play expressively. In our experiments, we have used Real Book jazz ballads. This work has been awarded the ńSwets & Zeitlingerż prize of the International Computer Music Association. This is the most prestigious award in the field of computer music.

#### Automated deduction in generalized possibilistic logic

Possibilistic logic is a logic of uncertainty that has many applications to plausible reasoning under incomplete information. Automated proof techniques were also developed for a classical first order language. Things become much more complex (both semantically and syntactically) when one allows the language to deal with imprecise or fuzzy constants, a very natural extension. Therefore, a line of research was en developed in order to provide both semantic foundations and efficient and sound proof methods. Some interesting results were obtained, and two different extended possibilistic logic programming systems PLFC and PGL were proposed and fully investigated.

### RECENT CONTRIBUTIONS: 2002-2014

During this last period we have continued to play a key international role in the definition and development of Mathematical Fuzzy logic and we have obtained important results in the following topics: 1) General and deep results for completeness of fuzzy logics either propositional or first order with respect to different semantics (real, hyper-real, rational

and finite) that cover and significantly extend previous results in the field. Our results have been possible as a consequence of a fruitful collaboration with researchers from different leading institutions on the topic; 2) Formalization of t-norm based logics dealing with partial degrees of truth, with algebraic semantics, and axiomatization and completeness results, both for propositional and first order languages, which have high applicability in modeling graded notions; 3) Development of different systems of fuzzy modal logic, with applications to reasoning under different forms to uncertainty on nonBoolean algebras of events; and 4) Development of a new hierarchy of Fuzzy Description logics, along

with new complexity results based on results of Mathematical Fuzzy Logic.

Moreover, in this last period, in collaboration with leading international researchers in the area of computational argumentation, we have also extended the computational argumentation of Defeasible Logic Progamming (DeLP), with the treatment of possibilistic uncertainty at object level, allowing to stratify defeasible rules in a DeLP program according to their strength, and by defining a new recursive semantics which avoids some undesired side effects of the original semantics based on dialectical trees.



### RECOGNITION

### Homage to Elie Sanchez

Bernadette Bouchon-Meunier



Elie Sanchez, picture taken by E.H. Mamdani in 2006

It is with great sadness that we pay tribute to the memory of Elie Sanchez, who passed away on March 6, 2014.

Elie Sanchez was a pioneer in fuzzy sets and systems, not only in France where he lived and where he was one of the founders of the fuzzy community, but also in the world.



E. Sanchez and A. Kaufmann in 1980

After a PhD in Mathematics, he defended a Dr. Sci. thesis in Human Biology in 1974, entitled "Equations de relations floues" (Fuzzy relation equations) with a proposed application to medical diagnosis assistance, launching applications

of fuzzy sets in medicine, as well as theoretical advances of fuzzy set theory. He collaborated with Arnold Kaufmann, the author of the first textbook on fuzzy sets in France in 1973, translated into English in 1975, and he published books with him.



L.A. Zadeh and E. Sanchez in New Orleans in 1977

He wrote a paper on "Resolution of eigen fuzzy sets equations", published in the very first issue of the journal Fuzzy Sets and Systems, in which Lotfi A. Zadeh also published one of his seminal papers on "Fuzzy sets as a basis for a theory of possibility", in 1978. A Visiting Research Associate at the University of California at Berkeley in 1976-1977 and 1981-1982, Elie Sanchez started a collaboration with Lotfi A. Zadeh and they later co-edited a first book on "Approximate reasoning in intelligent systems, decision and control" published by Pergamon Press in 1987, and a second one on "Genetic algorithms and fuzzy logic systems" published by World Scientific in 1997, proving that he was very early aware of the importance of so-called hybrid systems, taking advantage of several methodologies of computational intelligence.



M. Mizumoto, E. Sanchez, M. Umano, D. Willaeys in Marseille in 1978

In addition to all the books on fuzzy set theory and applications he co-edited from 1982, he was also very active in the development of the fuzzy logic community through

the organization of one of the very first conferences dedicated to this topic, the International Colloquium on the theory and applications of fuzzy sets, held in Marseille in 1978. He also chaired the IFAC Symposium on Fuzzy information, knowledge representation, and decision analysis in Marseille in 1983.



T. Yamakawa, E. Sanchez and L.A. Zadeh in Kumamoto in 1989

He was invited in many countries, for instance in Canada for the first time in 1977, in China from 1981, in Japan from 1984, in Australia from 1996. In all these countries, he collaborated with leaders in fuzzy sets, gave lectures and published joint papers. He started to participate in the international development of fuzzy control in 1989, publishing papers and collaborating with companies such as PSA in France

or NASA Ames Research Center in USA. He built the Neural & Fuzzy Systems Institute in Marseille, to both manage industrial collaborations and develop theoretical advancements, inviting foreign researchers to participate.

He was recognized worldwide, chairing the International Fuzzy Systems Association from 1989 to 1991 and being awarded an IFSA fellow in 2001.

He was a Council Member or a Visiting Research Associate of several institutes in Japan, China, Argentina, USA, Switzerland. He received the International MOISIL Gold Medal and Award in 1995. Elie Sanchez will be remembered for his scientific achievements, his charisma and his exceptional human qualities.



M. Sugeno, J.L. Verdegay, M.T. Lamata, B. Bouchon-Meunier, E. Sanchez at IPMU 2006 in Paris

### JOURNAL HIGHLIGHTS

### Transactions on Fuzzy Systems

The IEEE Transactions on Fuzzy Systems (IEEE TFS) publishes high quality technical papers in the theory, design and application of fuzzy systems. Readers are encouraged to submit papers which disclose significant technical knowledge, exploratory developments and applications of fuzzy systems. Emphasis is given to engineering systems and scientific applications. The Transactions also contains a letters section which includes information of current interest, and comments and rebuttals submitted in connection with published papers.

This journal has focused on following subjects and its applications:

- Aerospace
- Components, Circuits, Devices & Systems
- Computing & Processing (Hardware/Software)
- Engineering Profession
- · Fields, Waves & Electromagnetics
- General Topics for Engineers (Math, Science & Engineering)

- Power, Energy, & Industry Applications
- Robotics & Control Systems
- Signal Processing & Analysis
- Transportation

Attention paying on this journal has increased greatly in recently years. The submission of TFS has increased tremendously from 377 (in 2010) to 781 (in 2013). The acceptance rate is about 19% (in 2013). Many researchers, scholars and fellows found that most papers published by TFS are very innovative and constructive. The Impact Factor (IF) of TFS has increased from 2.695 (in 2010) to 6.306 (in 2013). The IF of TFS in 2013 ranked number 2 out of 121 publications in the CS/AI Categories, ranked number 8 out of 142 publications among all IEEE Publications, and ranked number 3 out of 247 publications in the EEE Categories.

Please do not hesitate to contact the editors if there is any question or inquiry.

Chin-Teng Lin (Taiwan) Editor in Chief

More information at http://cis.ieee.org/ieee-transactions-on-fuzzy-systems.html

### **ACTIVITY REPORT**

## **EUSFLAT Working Group on Image Processing** is again active

### Irina Perfilieva

Let me start this short contribution with the following description of the field:

"Image Processing is among rapidly growing technologies today, with applications in various aspects of a human life. Image Processing became a core research area within engineering and computer science disciplines. It is focused on improving a quality of raw data from imaging sensors, getting over visible flaws and restoring originality of information. We believe that with increasing sophistication and power of the modern computing, computation will go beyond conventional, Von Neumann sequential architecture and would contemplate the optical execution too."

On the last EUSFLAT Assembly in Montpellier, it has been announced that the registered Working Group on Image Processing is not active anymore. As a reaction on that critical remark, several active members took initiative and organized a meeting on the FLINS conference in Joao Pessoa in August, 2014. Besides others, the meeting was attended by Etienne Kerre (one of previous coordinators), who informed everybody about the history of the Working Group, its main achievements and explained why the group lost visibility.

Since the beginning (at the year 2002), the Working Group was concentrated around the young researchers of the Ghent University and was headed by Prof. Kerre and administrated by Dr. Mike Nachtegael. Their main efforts were concentrated on organizing special sessions on the most important conferences of our society, including IFSA, IPMU and FUZZ-IEEE. They also organized several special issues of the journals of Soft Computing and of Approximate Reasoning and one edited volume "Soft Computing in Image Processing: Recent Advances", Vol. 210, that was published in the series "Studies in Fuzziness and Soft Computing" of Springer. However, after the retirement of Prof. Kerre (at the year 2010) and another engagement of Dr. Nachtegael, the Working Group on Image Processing did not show signs of active life

On the meeting in Joao Pessoa, it has been agreed that the following four EUSFLAT members will take responsibility as new coordinators: Prof. H. Bustince, Prof. E. Kerre, Prof. J. Montero and Prof. I. Perfilieva, and the activity of the Working Group will be relaunched. In September 2014, the EUSFLAT board approved the initiative and the program.

During the first three months the following steps have been attempted:

- the web page has been created, please visit it using http://www.eusflat.org/research\_wg\_ scip.php our sincere thanks for this job is addressed to Peter Hurtik, Ostrava (contact him with Petr.Hurtik@osu.cz for various related proposals);
- two special sessions have been proposed and ap-

proved; they are:

- Image Processing with Fuzzy Techniques on the 2015 IFSA/ EUSFLAT World Congress that will be held in Gijón, Asturias, Spain in June 30 - July 3, the invitation can be found at http://www.softcomputing.es/ metaspace/portal/12/487;
- Image Processing using Fuzzy Techniques on 2015 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2015) that will be held in Istanbul, Turkey. August 2 - 5, the invitation can be found at http://fuzzieee2015. org/wp-content/uploads/2014/10/ A-Special-Session-FUZZIEEE2015-humberto2. pdf;

We sincerely invite everybody who is interested in contributing to this topic and wants to discuss with involved colleagues their ideas and results (even preliminary).

At the end of this short message I would like to introduce two local groups - members of the EUSFLAT Working Group on Image Processing.

The first group is that to which I belong - a part of the Institute of Fuzzy Modelling and Applications (Ostrava University). It has 6 members: Peter Hurtík, Petra Hod'áková, Irina Perfilieva, Marek Vajgl, Pavel Vlašánek, Nicolas Madrid. Our challenge is to put all image processing tools on the platform of Soft Computing and in particular, using the technique of the F-transform. Other developing techniques are: image fusion, reconstruction, handling of fuzzy images, fuzzy associative memories, mathematical morphology. We are ambitious to do everything efficiently and to overperform classical algorithms. We are involved into several projects (republic level) - non-invasive control of jewelry stones, assistive medicine, mobile gadgets for easy driving. We are looking for partners to apply for a European project. The coordinator of this group is the author of this contribution - Prof. Irina Perfilieva http://irafm.osu.cz/en/c29\_ irina-perfiljeva-csc-prof with the e-mail address Irina.Perfilieva@osu.cz.

The second group "Fuzzy Logic and Information Fusion" (LOBFI) belongs to the University of the Balearic Islands (UIB) in Palma (Spain). The group unifies the following researches: Daniel Ruiz Aguilera, Manuel González Hidalgo, Arnau Mir and Sebastia Massanet. It studies, among other lines of research, the application of fuzzy logic techniques in image processing and analysis. In particular, the fuzzy mathematical morphology based on fuzzy conjunctions such as tnorms and uninorms, and fuzzy implications, and its application to edge detection, noise reduction, pattern recognition and object detection is now extensively studied. The main

achieved results have been presented in many specialized conferences and published in international journals in the field. Recently, the LOBFI group has become a founder member of the Health Research Institute of Palma (IdISPa) and it is now in touch with several clinical research groups. From these contacts with medical professionals, the group is focusing now in biomedical image processing and the application of fuzzy techniques in medical problems such as melanoma detection and segmentation, retinal vessel extraction, image fusion from several sources (MRI, PET, TAC), among others. The coordinator of this group is Dr. Sebastia Massanet http://www.uib.es/es/personal/ABjEwNzczNA/.

At the very end I announce the direct link where you may register as a member and join our EUSFLAT Working Group on Image Processing:

http://graphicwg.irafm.osu.cz/index.php?title=Special:UserLogin&returnto=Main+Page&type=signup

Please, send us your information, suggestions, critics, announcements, photos - we will be glad to communicate with you on every level.

Yours sincerely Irina Perfilieva

## Grade in Metalogical Notions: a Comparative Study of Fuzzy Logics

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Abstract. No new theorems either within a logic or of some metalogic are proved in this paper, but some already existing logic systems of fuzzy logic in the narrow sense are only rewritten. In so doing three levels of languages, namely object level, meta level, meta meta level, are distinctly presented obeying the principles of 'use' and 'mention'. The purpose of such a rewriting is to show that association of grades to the metalogical notions e.g. consistency, consequence etc. is properly accomplished only in the approach of graded consequence. A comparison with other fuzzy logics, primarily with that of Pavelka has been carried out from this angle.

**Keywords**. Fuzzy logic, Graded consequence, Metalanguage, Metalogical concepts, 'Use' and 'Mention' of a symbol.

#### 1. Introduction

In one of the papers by Pelta et.al in Mathware and soft computing [31], the authors made a remark: "Until now the construction of superficial many-valued logics, that is, logics with an arbitrary number (bigger than two) of truth values but always incorporating a binary consequence relation, has prevailed in investigations of logical many-valuedness". This remark is a genuine observation. However, in 1986 one of the present authors addressed precisely this issue and made a proposal to lift many-valuedness to metalogical notions. Subsequently some other work [5, 7, 8, 9] was done based on the idea presented in [6]. Prior to this, fuzzy logic was magnificently developed by Pavelka [29] and others. Apparently it may seem that fuzzy logic deals with many-valuedness in the notion of consequence. The main objective of this paper is to show that this is not the case. On the other hand, the notion of graded consequence has taken up this issue seriously and has developed the formalism to a considerable length.

A secondary objective of this work is to make it transparent that in a logic-discourse actually three levels of activities take place. This distinction is clearly visible in the graded consequence approach to reasoning with vagueness. The present scrutiny of the informal metatheory of the fuzzy logics paying proper attention to this distinction of levels reveals that some notions which appear to be 'meaningful' in those logics are not really 'well-founded'. This is because the principles of 'use' and 'mention' for maintaining the level distinction are not properly adhered to in these cases.

The language (formal) of a logic (formal) consists of well formed formulae (wffs), some finite strings of symbols taken from a set called the alphabet of the logic. This constitutes the object language. But logic-activity does not comprise of formation of wffs only. It also includes derivating or deriving and which forms the main part of it. This means writing of a finite sequence of wffs following certain 'principles', and using a set of wffs, called the premise. Thus although members of the object language are entities of the object level (or level-0) these are not the sole constituents of this level. The other entities are sets of wffs, fuzzy sets of wffs, finite sequences of wffs, the truth values of the wffs etc. Some of these are 'used' at level-0. The act of derivation is a level-0 activity in which a sequence of wffs is used. Deliberation over level-0 entities require a language in which these entities are 'mentioned'. The language is known as 'metalevel language', and the layer of activity may be called the metalevel activity. We may denote this level as level-1. The importance of discerning levels in the study of formal theories by making explicit the 'use' and 'mention' of symbols has been focused by many [11, 22, 23]. Making this distinction in the proper way, in a particular case of fuzzy logic, viz. Pavelka logic and graded consequence shall play a key role in the present study. Metalevel language for a formal logic deals with sentences like, 'a formula is a syntactic/semantic consequence of a set of formulae', 'a set of formulae is consistent', 'a formula is a theorem', 'a formula is a tautology' etc. So the notions of consequence, consistency, theoremhood, tautologihood, etc. are the constituents of the metalanguage of the logic. These are entities of level-1.

Liberalizing the rigidity of two-valuedness of logic, manyvalued logics took an entry into the scenario in early twenties of the past century. There had been both philosophical as well as mathematical motivations behind the emergence of many-valuedness. Sentences or wffs are assigned values other than true(1) or false(0) - the set of values in the new context being finite or infinite, linearly ordered or lattice ordered, usually there being the top and the least elements, denoted by 1 and 0 respectively. In all many-valued logics, although the wffs at the object levels are allowed to have values other than the top and the least, the metalevel sentences are two-valued. 'The set X of wffs is consistent' is either true or false, and the set of sentences at level-1 obey usually the rules of classical logic or at the most intuitionistic two-valued logic.  $X \vdash \alpha$  or  $\alpha$  is a syntactic consequence of X iff certain crisp rules hold.  $X \models \alpha$  or  $\alpha$  is a semantic consequence of X iff certain crisp conditions are satisfied e.g., for every valuations v, if v assigns designated values to the wffs in X then it has to assign some designated value to  $\alpha$  also. Thus Pelta's remark as mentioned at the beginning of this article is cor-

We would like to emphasize that while being engaged in logic-act, one has to use one more level too viz. the metametalevel, that talks about metalevel concepts. For example to define the notion of consequence which is an entity of level-1, a meta-metalevel language is sought for. Similar is the case while establishing the relationship between the level-1 notions viz. consistency and consequence. These may be categorized as level-2 activities that require sentences with a different set of predicates, functions etc. In case of classical two-valued logic all the three levels coalesce. In case of classical many-valued logics only two levels viz. levels 0 and 1 are visible. In fuzzy logics a.la Pavelka and others [20, 26, 27], the levels are blurred while in the theory of graded consequence [5, 6, 7, 8, 9] one can clearly distinguish all the three levels. It is interesting to note that in the theory of graded consequence sentences of levels 0 and 1 are many-valued (in general) while sentences of level-2 are two-valued. In this study, efforts shall be made to make the scenario of fuzzy logic clearer which is as yet blurred and it will ultimately be noticed that the metalevel sentences in fuzzy logics are not many-valued at all. Thus fuzzy logic is not genuinely dealing with degree of derivability or degree of consistency, but claims have been made in that respect on some occasions [29] as will be quoted later in section 5.1. One aim of this essay is to refute this claim by contrasting Pavelka logic with the approaches in graded consequence, and using methods of discerning the levels 0, 1, and 2 by using quotation marks ' ' for level-1, and " " for level-2. In section 4 and 5, while rewriting the theory of graded consequence and existing fuzzy logics we use

'' formally. But informal use of this symbol can also be found in other sections.

The present paper is a continuation to, and improvement over our earlier papers [6, 7, 8, 9]. It is known that Hajek in [21] has presented a version of Pavelka logic that we shall henceforth refer to as Pavelka-Hajek logic which Hajek has called Rational Pavelka Logic. A section 5.5 of this article is devoted to a scrutiny of this version in order to check if it suggests any way to see Pavelka logic as addressing many-valuedness at level-1. Finally in section 6, a discussion shall be initiated on the relative merits of graded consequence approach over the other fuzzy logics in respect of 'freedom' enjoyed by the doer of logic and perhaps in respect of application too.

## 2. Notion of Consequence: An initial comparison between Pavelka approach and the theory of graded consequence

A language with a notion of consequence constitutes a logic. So the notion of consequence is one of the basic concepts of logic. Tarski introduced the notion of consequence as an operator which maps a set of formulae to its consequence set, satisfying certain conditions [33]. On the other hand Gentzen presented the notion of consequence as a binary relation between two finite sequences of formulae and postulated by some conditions [15]. The two notions are equivalent in the cases of some logics e.g. classical logic and intuitionistic logic. Pavelka [29] in 1979 generalized Tarski's notion of consequence in the fuzzy context. In this context the notion of consequence is an operator *C* over the set of all

fuzzy subsets of formulae  $\mathcal{F}(F)$ , F being the set of formulae, and C satisfies the following conditions,

- $X \subseteq C(X)$ ,
- if  $X \subseteq Y$  then  $C(X) \subseteq C(Y)$ ,
- $\bullet \ C(C(X)) = C(X).$

In Pavelka's approach the notion of inclusion between two fuzzy subsets has been understood as a crisp relation viz., for two fuzzy subsets X and  $Y, X \subseteq Y$  iff for all  $\alpha$  in  $F(X(\alpha))$ , the membership degree of  $\alpha$  in X, is less or equal to  $Y(\alpha)$ , the membership degree of  $\alpha$  in Y.

Chakraborty in 1986, offered a generalization to a slightly modified (but equivalent) version of Gentzen's notion of consequence in the fuzzy case in a technical report subsequently published in 1988 [6]. In many-valued context a consequence relation, called graded consequence relation, is a fuzzy relation from an ordinary subset of formulae to a single formula, satisfying,

- if  $\alpha \in X$  then  $gr(X \mid \sim \alpha) = 1$  (Reflexivity),
- if  $X \subseteq Y$  then  $gr(X \mid \sim \alpha) \le gr(Y \mid \sim \alpha)$  (Monotonicity),
- $inf_{\beta \in \mathbb{Z}} \operatorname{gr}(X \mid \sim \beta) \wedge \operatorname{gr}(X \cup Y \mid \sim \alpha) \leq \operatorname{gr}(X \mid \sim \alpha)$  (Cut)

where ' $\leq$ ' and ' $\wedge$ ' are the order relation and the corresponding meet operation of the lattice structure for the truth set of the metalogic.  $\operatorname{gr}(X \mid \sim \alpha)$  is an element of this lattice denoting the grade of relatedness of  $\alpha$  to X. For the sake of doing the comparison of Pavelka logic with graded consequence, in the present paper, this structure is considered as a complete pseudo Boolean algebra (Heyting Algebra). In [7, 9] more generalized structures are taken.

The distinguishing features of the two approaches are the following.

- 1. In the first approach the notion of consequence has been generated through a crisp operator C from  $\mathscr{F}(F)$  to  $\mathscr{F}(F)$ . Whereas, in the second approach it has been defined as a fuzzy relation from P(F), the power set of F to F. More precisely, in the first approach a fuzzy set of formulae is assigned to another fuzzy set of formulae and is called the consequence set of the latter. But in the second approach the premise (X) is always crisp, the conclusion  $(\alpha)$  is a well-formed formula but there is a degree of derivability of the wff from the premise viz.  $\operatorname{gr}(X \mid \sim \alpha)$ .
- 2. An investigation by Gerla [16] reveals that, these two notions are not equivalent.

### 3. Metalogical notions in the theory of graded consequence vis-a-vis the classical perspective

To clarify the process of assigning grades to the metalogical notions, let us first consider all the necessary metalogical concepts and their formalized versions in the theory of graded consequence, and contrast them with their respective counterparts in two-valued situation.

**3.1 Semantic consequence** Firstly let us consider the metalogical statement ' $\alpha$  is a semantic consequence of X'. Classical two-valued understanding of the above statement is, 'for all states of affair T given by a valuation function, if

X is contained in T then  $\alpha$  is a member of T, where T is identified with the set of formulae which are true (1) under the valuation function T. A formal expression of the above statement would be  $\forall T(X \subseteq T \to \alpha \in T)$ , abbreviated by  $X \models \alpha$ , where all the notions are standard set theoretic, and ' $\rightarrow$ ' stands for the metalevel connective 'if-then'.

This idea of semantic consequence was generalized by Shoesmith and Smiley [32] in 1978. According to [32], T needs not range over all states of affair. That is, given an arbitrary collection of states of affair, a subset of P(F),  $\alpha$  is a semantic consequence of X iff for all states of affair T in that collection, if X is contained in T then  $\alpha$  is a member of T.

In two-valued situation, one can have only two possibilities i.e. either ' $\alpha$  is a semantic consequence of X' receives the value true or receives the value false. At this very point, theory of graded consequence has taken a deviated view, by offering a many-valued reading to the sentence ' $\alpha$  is a semantic consequence of X'. Furthermore, it offers a reasonable process of computing the truth value of the sentence just by lifting the classical version to the many-valued situation. That is, the sentence ' $\alpha$  is a semantic consequence of X', denoted as  $X \approx \alpha$ , would get the value  $gr(X \approx \alpha)$ , given by.

 $\operatorname{gr}(X|\approx \alpha)=\inf_i[\operatorname{gr}(X\subseteq T_i)\to \alpha\in T_i],$  where  $\{T_i\}_{i\in I}$  is an arbitrary collection of fuzzy subsets over F. The lattice ' $\inf$ ' and the fuzzy implication operator ' $\to$ ' are being used to compute the universal quantifier ' $\forall$ ' and the connective 'ifthen' respectively. The truth-value lattice for the metalevel is taken to be complete. It is to be noted that states of affair are now fuzzy, which are valuations in an appropriate truth value set for the wffs at level-0. But the structure of the truth-value set for wffs is not necessarily the same as the structure of the truth-value set for the level-1 sentences. It is also to be noted that the inclusion relation ( $\subseteq$ ) is fuzzy too. That is for any set X and fuzzy subset  $T_i$ ,  $\operatorname{gr}(X\subseteq T_i)$  is computed by  $\inf_{x\in X}[\operatorname{gr}(x\in X)\to\operatorname{gr}(x\in T_i)]=\inf_{x\in X}[1\to T_i(x)]=\inf_{x\in X}[T_i(x)].$ 

**3.2 Tautology** Classically,  $\alpha$  is a tautology if and only if any valuation gives it the value 1. Generalizing in the line of Shoesmith and Smiley, given a collection  $\{T_i\}_{i\in I}$  of valuations or equivalently fuzzy sets of formulae, ' $\alpha$  is a tautology' means  $\alpha \in T_i$  for all  $i \in I$ . In the theory of graded consequence the value (i.e. the tautologihood degree of  $\alpha$ ), proposed for the sentence ' $\alpha$  is a tautology' is  $\mathrm{Taut}(\alpha)$ , and given any arbitrary but fixed collection of fuzzy subsets  $\{T_i\}_{i\in I}$  over F,  $\mathrm{Taut}(\alpha)=\inf_i T_i(\alpha)$ .

**3.3Derivation** To claim 'A finite sequence of formulae  $\alpha_1,\alpha_2,\ldots,\alpha_n$  is a derivation of  $\alpha_n$  from a set of formulae X' classically, one needs to pass through the process viz. whether, for each i,  $1 \le i \le n$ ,  $\alpha_i \in X$  or  $\alpha_i$  is an axiom or  $\alpha_i$  is a conclusion of an instance of a rule whose premises occurred in the sequence preceding  $\alpha_i$ . After carrying out this process successively along the whole sequence, one can determine ' $\alpha_1,\alpha_2,\ldots,\alpha_n$  to be a derivation of  $\alpha_n$  from X', or symbolically ' $(\alpha_1,\alpha_2,\ldots,\alpha_n)$  D(X,  $\alpha_n$ )'. The value attached to this sentence  $(\alpha_1,\alpha_2,\ldots,\alpha_n)$  D(X,  $\alpha_n$ ) is val( $(\alpha_1,\alpha_2,\ldots,\alpha_n)$  D(X,  $\alpha_n$ )), which is given by val( $(\alpha_1,\alpha_2,\ldots,\alpha_n)$  D(X,  $\alpha_n$ )) =  $|\alpha_1| \wedge |\alpha_2| \wedge \ldots \wedge |\alpha_n|$ , where for each i,  $1 \le i \le n$ 

$$\mid \alpha_i \mid = \left\{ \begin{array}{ll} 1 & \text{if } \alpha_i \in X \\ \text{Taut}(\alpha_i) & \text{if } \alpha_i \in A \backslash X \text{, where } A (\subseteq F) \text{ is a set of axiom} \\ \mid R_j \mid & \text{if } \langle \alpha_{i_1}, \alpha_{i_2}, \ldots, \alpha_{i_j}, \alpha_i \rangle & \text{belongs} \\ & \text{to the support of the fuzzy} \\ & \text{rule of inference } R_j, \text{ which is} \\ & \text{a fuzzy relation from } F^j \text{ to } F \\ & \text{, } i_1, i_2 \ldots, i_j < i, \text{ and } \mid R_j \mid = \\ & R_j(\langle \alpha_{i_1}, \alpha_{i_2}, \ldots, \alpha_{i_j} \rangle, \alpha_i), \text{ the degree to which } \langle \alpha_{i_1}, \alpha_{i_2}, \ldots, \alpha_{i_j} \rangle \text{ is} \\ & R_j\text{-related to } \alpha_i. \end{array} \right.$$

- **3.4 Syntactic consequence** Classically, ' $\alpha_n$  is a syntactic consequence of X' means that there is a sequence of formulae  $\alpha_1, \alpha_2, \ldots, \alpha_n$  such that  $\alpha_1, \alpha_2, \ldots, \alpha_n$  is a derivation of  $\alpha_n$  from X. Following the above idea, the value  $\sup val((\alpha_1, \alpha_2, \ldots, \alpha_n)D(X, \alpha_n))$  is assigned to the sentence ' $\alpha_n$  is a syntactic consequence of X', where  $\sup val = \sup val$
- **3.5 Consistency** Classical understanding of 'X is consistent' is that X has a model; more formally,  $\exists T_i(X \subseteq T_i)$ , where  $\{T_i\}_{i\in I}$  is the set of some valuations (after extending according to [32]). In the theory of graded consequence, given  $\{T_i\}_{i\in I}$ , a collection of fuzzy subsets over F, the sentence 'X is consistent' would have some value (not necessarily true (1) or false (0)), computed by  $CD(X) = \sup_i [gr(X \subseteq T_i)]$ . CD(X) is read as the consistency degree of X, where ' $\sup$ ' is used to compute the existential quantifier ' $\exists$ '.
- **3.6 Theoremhood** Similarly, ' $\alpha$  is a theorem' is given values by generalizing the idea ' $\alpha$  is a syntactic consequence of the empty set' (or ' $\alpha$  is provable').
- **3.7 Inconsistency** 'X' is inconsistent' is defined by 'any formula is a syntactic consequence of X'. So, inconsistency degree of a set of formulae X is computed by  $inf_{\alpha} gr(X \mid \sim \alpha)$ . Thus consistency degree and inconsistency degree are measured from two angles. The former from the semantic and the latter from the syntactic angle. It turns out that 'X' is inconsistent' is a different notion in level-1, and it is not simply the negation of 'X' is consistent'.
- 3.8 Soundness, Completeness Soundness and completeness of a logical system deal with the interrelation between the notion of syntactic consequence and the notion of semantic consequence. These notions can be viewed in two different ways -(1) as notions of metalevel, (2) as notions of meta-metalevel. At metalevel the notion of soundness is expressed as 'F or all X, for all  $\alpha$ , if  $\alpha$  is a syntactic consequence of X then  $\alpha$  is a semantic consequence of X' and the notion of completeness is expressed as 'F or all X, for all  $\alpha$ , if  $\alpha$  is a semantic consequence of *X* then  $\alpha$  is a syntactic consequence of X'. Now classical two-valued understanding of syntactic consequence, semantic consequence and the connective 'ifthen' render two-valuedness to the notions of soundness, completeness. But in the theory of graded consequence one has the option open for giving them a many-valued understanding too i.e. the notions of soundness or completeness

would be graded if one allows to compute the connective 'ifthen' by a fuzzy implication operator. On the other hand, at meta-metalevel, soundness and completeness are expressed as relations between 'syntactic consequence' and 'semantic consequence'. The existing theory of graded consequence proposes soundness and completeness as two-valued notions, while keeping the provision open for a many-valued interpretation, as mentioned above.

### 4. Rewriting the theory of graded consequence maintaining level-distinction

Although the theory of graded consequence was initiated in 1986, and it was clear from the begining that it would be possible to write the theory by formally showing the three levels of sentences, in this paper we present the levels explicitly.

#### Level-0 consists of:

- Formulae  $\alpha$ ,  $\beta$ ,  $\gamma$  ...with or without superscripts and subscripts.
- All ordinary sets of formulae  $A_X$ , X, Y, Z,...
- A collection of fuzzy sets of formulae  $A_X$ ,  $T_1$ ,  $T_2$ ,  $T_3$ ,...
- All finite sequences of formulae  $\langle \alpha_1, \alpha_2 ..., \alpha_n \rangle$ ,  $\langle \beta_1, \beta_2 ..., \beta_m \rangle$ , ...
- Some special sets of finite sequences of formulae of the same form, called the syntactic components of the rules of inference,

$$\begin{split} S_i^1 &= \{\langle \alpha_1^j, \alpha_2^1, \dots \alpha_i^1, \alpha^1 \rangle, \langle \beta_1^1, \beta_2^1, \dots \beta_i^1, \beta^1 \rangle, \dots \} \\ \vdots \\ S_m^j &= \{\langle \alpha_1^j, \alpha_2^j, \dots \alpha_m^j, \alpha^j \rangle, \langle \beta_1^j, \beta_2^j, \dots \beta_m^j, \beta^j \rangle, \dots \} \end{split}$$

#### Note 4.1

- 1.  $A_X$  is a special set of formulae, called logical axioms.
- 2.  $T_1$ ,  $T_2$ ,  $T_3$ , ... are fuzzy subsets of formulae, identified with the truth valuation functions such that truth value of a formula under a valuation function is exactly same as the membership degree of that formula to the respective fuzzy subset. These are the relevant states of affair.
- 3.  $\widetilde{A}_X$  is a special fuzzy subset of formulae, identified with  $A_X \cap (\cap_i T_i)$  i.e.

$$\widetilde{A}_X(\alpha) = \bigcap_i T_i(\alpha)$$
 if  $\alpha$  is a member of  $A_X$ .  
= 0 otherwise.

4. Each of  $S_i^1, \ldots S_m^j$  contains finite sequences of formulae of the same form. What we intend to mean by the phrase 'finite sequences of formulae of the same form' is enunciated by an example:

If we consider  $S_2^1$  as the rule Modus Ponens then  $S_2^1$  would be the set

 $\{\langle \alpha, \alpha \supset \beta, \beta \rangle, \langle \gamma, \gamma \supset \delta, \delta \rangle \dots \}$  i.e. all instances of the rule. The subscript 2 denotes the arity of the rule, and the superscript denotes the enumeration number of the rule with arity 2.

#### Level-1 consists of:

- Names of some of the level-0 entities, viz.

'
$$\alpha$$
', ' $\beta$ ', ' $\gamma$ ' ... ' $X$ ', ' $Y$ ', ' $Z$ ',..., ' $\widetilde{A}_{X}$ ', ' $\langle \alpha_{1}, \alpha_{2}, \ldots, \alpha_{n} \rangle$ ', ' $\langle \alpha_{1}, \alpha_{2}, \ldots, \alpha_{m} \rangle$ ', ..., ' $S_{i}^{1}$ ', ..., ' $S_{m}^{j}$ '.

All these are constant symbols of level-1 language whose interpretations are the respective level-0 objects.

- Variables:
- 1. The formula variable x;  $x_1^1$ ,  $x_2^1$ , ...,  $x_i^1$ ,  $x^1$ ; ...;  $x_1^j$ ,  $x_2^j$ , ...,  $x_m^j$ ,  $x^j$  (range over all formulae of level-0).
- 2. The crisp set variable V (ranges over all ordinary sets of formulae of level-0).
- 3. The fuzzy set variable T (ranges over  $T_1$ ,  $T_2$ ,  $T_3$ , ... of level-0).
- 4. The sequence variable  $\langle \ \rangle_{SV}$  (ranges over all finite sequences of formulae of level-0).

5. 
$$(x_1^1, x_2^1, ..., x_i^1, x^1)$$
 (ranges over  $S_i^1$ ).

$$\vdots \\ \langle x_1^j, x_2^j, \dots, x_m^j, x^j \rangle \text{ (ranges over } S_m^j).$$

#### Note 4.2

- (1)  $\langle x_1^1, x_2^1, \dots, x_i^1, x^1 \rangle$ ,  $\ldots \langle x_1^j, x_2^j, \dots, x_m^j, x^j \rangle$  are used as some kind of sequence variables which range over some particular types of sequence of formulae, and are formed by the formula variables  $x_1^1, x_2^1, \dots, x_i^1, x^1, \dots; x_1^j, x_2^j, \dots, x_m^j, x^j$ . (2) This is, hence a many-sorted language, variables ranging over different sets. It is not required to take infinitely many variables of any sort. Depending of different rules of inference, different sets of finitely many formula variables are needed to form a sequence of variables ranging over each of such rules.
- Function Symbols: dom, ran.
- Binary Predicate Symbols:  $\in$ , R,  $\vdash$
- Propositional Connectives:  $\rightarrow$ , &,  $\vee$ .
- Quantifiers: ∀, ∃.
- Other Symbols: (,).
- Terms: Constant symbols and variables are terms. Also the expressions of the following kinds are considered as terms.

$$\begin{aligned} \operatorname{dom}(\langle x_1^1, x_2^1, \dots, x_i^1, x^1 \rangle), \dots, \operatorname{dom}(\langle x_1^j, x_2^j, \dots, x_m^j, x^j \rangle). \\ \operatorname{ran}(\langle x_1^1, x_2^1, \dots, x_i^1, x^1 \rangle), \dots, \operatorname{ran}(\langle x_1^j, x_2^j, \dots, x_m^j, x^j \rangle). \\ [\operatorname{dom}(\langle x_1^1, x_2^1, \dots, x_i^1, x^1 \rangle) \text{ denotes the set consisting of the} \end{aligned}$$

[dom( $\langle x_1^1, x_2^1, \ldots, x_i^1, x^1 \rangle$ ) denotes the set consisting of the subsequences formed by the first i component of each sequence of  $S_i^1$  and  $\operatorname{ran}(\langle x_1^1, x_2^1, \ldots, x_i^1, x^1 \rangle)$  denotes the set consisting of those formulae which form the last component of each sequence present in  $S_i^1$ . The subscript and superscript of the variables represent that they correspond to  $S_i^1$ , the first rule of arity i.]

- Some basic wffs involving the predicate ∈:

$$\begin{array}{l} `\alpha' \in `X', \ \dots, \ `\alpha' \in \ T, \ \dots, \ x \in `X', \ \dots, \ x \in \ T, \\ \dots, `\alpha' \in `\widetilde{A}_X', \ x \in \langle \ \rangle_{SV}, \ x \in `\langle \alpha_1, \alpha_2, \dots, \alpha_n \rangle', \ \dots, \ x \in \\ \operatorname{dom}(\langle x_1^1, x_2^1, \dots, x_i^1, x^1 \rangle), \ \dots, \ \operatorname{ran}(\langle x_1^1, x_2^1, \dots, x_i^1, x^1 \rangle) \in \ T \\ \end{array}$$

- A few examples of compound wffs involving the predicate  $\epsilon$ -

1. 'X' 
$$\subseteq$$
 'Y'  $\equiv \forall x (x \in X' \rightarrow x \in Y')$ 

['X'  $\subseteq$  'Y' (read as 'X' is included in 'Y') is an abbreviation of the level-1 formula

 $\forall x (x \in `X' \to x \in `Y')$  which means that for all x if x is a member of `X' then x is a member of `Y'].

2. 'X' 
$$\subseteq T \equiv \forall x (x \in X' \rightarrow x \in T)$$
.

3. 
$$\operatorname{dom}(\langle x_1^1, x_2^1, \dots, x_i^1, x^1 \rangle) \subseteq T \equiv \forall x \ (x \in \operatorname{dom}(\langle x_1^1, x_2^1, \dots, x_i^1, x^1 \rangle) \to x \in T).$$

4.  $\langle \ \rangle_{SV} \subseteq (\langle \alpha_1, \alpha_2, \dots, \alpha_n \rangle) \equiv \forall x \ (x \in \langle \ \rangle_{SV} \to x \in (\langle \alpha_1, \alpha_2, \dots, \alpha_n \rangle).$ 

5. 'X' 
$$\models$$
 ' $\alpha$ '  $\equiv \forall T ('X' \subseteq T \rightarrow '\alpha' \in T)$ .

[ 'X'  $\models$  ' $\alpha$ ' (read as ' $\alpha$ ' is a semantic consequence of 'X') is an abbreviation of the level-1 formula  $\forall T$  ('X'  $\subseteq T \rightarrow$  ' $\alpha$ '  $\in T$ ) which means that for all T, if 'X' is included in T then ' $\alpha$ ' is a member of T ]

6. Taut(' $\alpha$ ')  $\equiv \forall T \ ('\alpha' \in T)$ .

[ Taut(' $\alpha$ ') (read as ' $\alpha$ ' is a tautology) is an abbreviation of the level-1 formula  $\forall T$  (' $\alpha$ '  $\in T$ ) which means that for all T, ' $\alpha$ ' is a member of T ]

7. Cons('X')  $\equiv \exists T \ ('X' \subseteq T)$ .

[ Cons('X') (read as 'X' is consistent) is an abbreviation of the level-1 formula

 $\exists T \ (X' \subseteq T)$  which means that there is some T such that X' is included in T.

- Some examples of basic wffs with the predicate  $\vdash$ :  $(X' \vdash (\alpha', X' \vdash x))$ .
- Compound wffs involving  $\vdash$ : Incons('X')  $\equiv \forall x ('X' \vdash x)$ .

Before introducing wffs, involving R, let us see, how rules of inferences are introduced in classical situation. Let us consider the classical rule, 'from  $\alpha$  and  $\alpha \supset \beta$  infer  $\beta$ '.  $(\alpha, \alpha \supset \beta, \beta)$  are object level formulae and  $\supset$  is object level connective 'if-then'.) This corresponds to the syntactic part of the rule which is an instruction of how to act having some special form of formulae at hand. But to realize the justification of the instruction, one appeals to its meaning i.e. in the present case, the justification of the above instruction is 'whenever  $\alpha$  is true and  $\alpha \supset \beta$  is true  $\beta$  is also true'. So in classical case, to present a rule we need two metalevel sentences one pertaining to the syntactic part, which is an ordinary relation, and the other pertaining to the semantic part. In the theory of graded consequence, rules of inference are introduced to be fuzzy relations. Now to justify these fuzzy rules of inference, we need following level-1 formulae:

$$\begin{split} R_i^1 &\equiv \forall \ \langle x_1^1, x_2^1, \dots, x_i^1, x^1 \rangle (\forall T (\operatorname{dom}(\langle x_1^1, x_2^1, \dots, x_i^1, x^1 \rangle)) \subseteq \\ T &\rightarrow \operatorname{ran}(\langle x_1^1, x_2^1, \dots, x_i^1, x^1 \rangle) \in T). \\ \vdots \\ R_m^j &\equiv \forall \ \langle x_1^j, x_2^j, \dots, x_m^j, x^j \rangle (\forall T (\operatorname{dom}(\langle x_1^j, x_2^j, \dots, x_m^j, x^j \rangle) \subseteq \\ T &\rightarrow \operatorname{ran}(\langle x_1^j, x_2^j, \dots, x_m^j, x^j \rangle) \in T). \end{split}$$

These are to take care of the semantic part of a rule. The syntactic counterpart is given by the following wffs,

- Atomic wffs involving the predicate *R*:

$$R$$
 ( $\langle \ \rangle_{SV}$ , ' $\alpha$ '),  $R$  (' $\langle \alpha_1, \alpha_2, \dots, \alpha_n \rangle$ ', ' $\alpha$ ').  
[  $R$  (' $\langle \alpha_1, \alpha_2, \dots, \alpha_n \rangle$ ', ' $\alpha$ ') is interpreted as ' $\alpha$ '.

[ R (' $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle$ ', ' $\alpha$ ') is interpreted as ' $\alpha$ ' is related to ' $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle$ ' by a rule of inference. ]

In classical logic, let us take as an instance rule Modus Ponens viz., from  $\alpha$  and  $\alpha \supset \beta$  infer  $\beta$ . This syntactic part of the rule is considered to be valid because of the semantic counterpart i.e. whenever  $\alpha$  is true and  $\alpha \supset \beta$  is true  $\beta$  is true gets the truth value true. This principle may be accepted for any other rule. In accordance with the above understanding R (' $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle$ ', ' $\alpha$ ') is interpreted by a fuzzy relation which assigns  $|R_i^j|$ , the truth value of the leve-1 formula  $R_i^j$  to R (' $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle$ ', ' $\alpha$ ') if (' $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle$ ', ' $\alpha$ ')  $\in S_i^j$  and 0 otherwise.

- Compound wffs involving the predicate *R*:

1.  $Res_R(\alpha_i) \equiv \exists \langle \rangle_{SV} (\langle \rangle_{SV} \subseteq (\alpha_1, \alpha_2, ..., \alpha_{i-1}) \& R (\langle \rangle_{SV}, (\alpha_i)).$ 

[  $Res_R(`\alpha_i")$  (read as ' $\alpha_i$ ' is a result of a rule of inference applied on a subset of ' $(\alpha_1, \alpha_2, \ldots, \alpha_{i-1})$ ', and the idea behind the formula is, there is some sequence included in the sequence of formulae ' $(\alpha_1, \alpha_2, \ldots, \alpha_{i-1})$ ' such that ' $\alpha_i$ ' is related to that sequence by a rule of inference.]

2.  $(\alpha_1, \alpha_2, ..., \alpha_n)'D(X', \alpha_n') \equiv (\alpha_1' \in X' \vee \alpha_1' \in \widetilde{A}_X') \otimes (\alpha_2' \in X' \vee \alpha_2' \in \widetilde{A}_X' \vee Res_R(\alpha_2')) \otimes ... \otimes (\alpha_n' \in X' \vee \alpha_n' \in \widetilde{A}_X' \vee Res_R(\alpha_n')).$ 

[' $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle$ 'D('X', ' $\alpha_n$ ') is read as ' $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle$ ' is a derivation ' $\alpha_n$ ' from 'X'. The intended meaning of the formula is (either ' $\alpha_1$ ' is a member of the premise 'X' or ' $\alpha_1$ ' is an axiom) and (either ' $\alpha_2$ ' is a member of 'X' or ' $\alpha_2$ ' is an axiom or ' $\alpha_2$ ' is a result of a rule of inference from a subset of ' $\langle \alpha_1 \rangle$ ') and ... and (either ' $\alpha_n$ ' is a member of 'X' or ' $\alpha_n$ ' is an axiom or ' $\alpha_n$ ' is a result of a rule of inference from a subset of ' $\langle \alpha_1, \alpha_2, \ldots, \alpha_{n-1} \rangle$ '). ]

3. 'X'  $\vdash_{AX}$  '\alpha'  $\equiv \exists \ \langle \ \rangle_{SV} \ (\langle \ \rangle_{SV} \ D \ ('X', '\alpha')).$ 

['X'  $\vdash_{AX}$  ' $\alpha$ '(read as ' $\alpha$ ' is a syntactic consequence of 'X') is an abbreviation of the level-1 formula  $\exists \ \langle \ \rangle_{SV} \ (\langle \ \rangle_{SV} \ D$  ('X', ' $\alpha$ ')) which means that there is a sequence of formulae which is a derivation of ' $\alpha$ ' from 'X'.

Interpretation of the level-1 language

**Truth set-structure** The value set for this level-1 language is a complete pseudo Boolean algebra  $(L, \land, \lor, \rightarrow_A, 0, 1)$ . Operators  $\land$  and  $\rightarrow_A$  that form an adjoint pair are used to compute the level-1 connectives & and  $\rightarrow$  respectively, and the lattice 'inf' and 'sup' are used to take care of the quantifiers ' $\forall$ ' and ' $\exists$ ' respectively.

### Determination of the truth values of the basic wffs

- 1. Formulae of the type ' $\alpha$ '  $\in$  'X', would get the value 1 if interpretation of ' $\alpha$ ' i.e.  $\alpha$  is a member of the set referred to by 'X' and 0 otherwise.
- 2. Let us consider the wffs of the type ' $\alpha$ '  $\in$  T. As the wff contains a variable, viz. T this wff does not have a truth value as such but after being bounded in presence of quantifiers i.e.  $\exists$  (' $\alpha$ '  $\in$  T) and  $\forall$  (' $\alpha$ '  $\in$  T), the values are  $sup_iT_i(\alpha)$  and  $inf_iT_i(\alpha)$  respectively.
- 3. Similar interpretation is applicable to the wffs of the type  $x \in {}^{\iota}X'$ ,  $x \in T$ .
- 4. The sentences of the type 'X'  $\vdash$  ' $\alpha$ ' would be interpreted by a fuzzy relation between the set of sets of formulae and the set of formulae. In fact, there can be many such predicate symbols  $\vdash_1$ ,  $\vdash_2$ , ..., at level-1, and their respective interpretations i.e. several fuzzy relations at the corresponding algebraic structure of level-1.
- 5. Value of the wffs of the type R (' $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle$ ', ' $\alpha$ ') would be determined by the fuzzy relation, defined earlier in this section. That is, value of R (' $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle$ ', ' $\alpha$ ') is  $|R_i^j|$ , for some  $R_i^j$ , if (' $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle$ ', ' $\alpha$ ')  $\in S_i^j$  and 0 otherwise.

So after the determination of the truth values of the basic wffs it is quite immediate to see how the compound wffs, some of which are our well-known metalogical concepts, are being offered a many-valued reading.

**Example** To elucidate, let us consider the sentence 'X'  $\models$  ' $\alpha$ ' which stands for

 $\forall \ T \ (X' \subseteq T \to `\alpha' \in T) \ \text{i.e.} \ \forall \ T \ (\forall x \ (x \in `X' \to x \in T) \to `\alpha' \in T).$  Given a collection of fuzzy subsets  $\{T_i\}_{i \in I}, \ T$  will range over the collection, and  $`\forall'$  and  $`\to'$  will be computed by `inf' and  $`\to_A'$  respectively. Hence L being a set consisting of truth values other than true (1) and false (0), the truth value of the sentence is  $inf_i \ (inf_{x \in X} \ (1 \to T(x))) \to T(\alpha)) = inf_i \ (inf_{x \in X} \ T(x) \to T(\alpha))$ . This may be a value other than 0 and 1.

#### Note 4.3

One point we would like to mention here. For simplicity, supported by the work of Shoesmith and Smiley, at level-0 we have taken one collection of fuzzy subsets, viz.  $T_1$ ,  $T_2$ ,  $T_3$ , ... instead of all fuzzy subsets over formulae, as is done in the classical fuzzy logics. But it is possible to incorporate several sets of fuzzy subsets of formulae at level-0. In that case we would take several collections of fuzzy subsets of formulae, i.e.  $\{T_i\}_{i\in I}$ ,  $\{T_j\}_{j\in J}$ , ... at level-0 and define the notion of semantic consequence, consistency, tautologihood etc. with respect to each of the collections of fuzzy subsets of formulae.

Level-2 consists of the following.

Alphabet: Constant symbols: "w", (where w is a wff of level-1.)

"\-", "\=", "\- $_{AX}$ ".
Predicate Symbol:  $\gg$ 

#### Note 4.4

It is to be noted that  $\models$  and  $\vdash_{AX}$  are defined symbols at level-1. We need to give names to these symbols too to talk about them at level-2. Analogy may be taken from classical logic. With  $\neg$  and  $\supset$  as primitive,  $\land$  and  $\lor$  are defined, and then we talk about conjunction (' $\land$ ') and disjunction (' $\lor$ ').

Terms: Constant symbols are terms.

Basic wffs:  $t_1 \gg t_2$ , where  $t_1$ ,  $t_2$  are names of level-1 formulae.

### Interpretation of level-2 language

- 1. Interpretation of each constant symbol of level-2 language is the corresponding level-1 entity. That is, the interpretation of "w" is the level-1 formula w, and that of " $\vdash$ ", " $\models$ ", " $\vdash_{AX}$ " are  $\vdash$ ,  $\models$ ,  $\vdash_{AX}$  respectively.
- 2. Interpretation of the predicate symbol  $\gg$  is such that the wff  $t_1 \gg t_2$  gets the value 1 (true) if the value of the formula referred to by  $t_1$  is less or equal to the value of the formula referred to by  $t_2$  and 0 otherwise.

Now in the metalanguage of level-1 language i.e. at level-2 we talk about the behaviour of the notion of consequence, a level-1 concept, by the following description.

Definition: A fuzzy relation "\—" is a graded consequence relation if "\—" satisfies,

- "' $\alpha$ '  $\in$  'X' " $\gg$ " 'X'  $\vdash$  ' $\alpha$ ' ",
- " 'X'  $\subseteq$  'Y' & 'X'  $\vdash$  ' $\alpha$ ' "  $\gg$  " 'Y'  $\vdash$  ' $\alpha$ ' ", and
- "  $\forall x \ (x \in {}'Z' \rightarrow {}'X' \vdash x) \& {}'X \cup Z' \vdash {}'\alpha' " \gg " {}'X' \vdash {}'\alpha' "$  for all wffs  $\alpha$  and sets X, Y, Z of wffs.

This definition constitutes the formal representation of the Gentzen type generalization of consequence relation presented in section 2. Also, it is to be noted that the expressions under the quote "" are metalevel (or level-1) sentences.

#### Note 4.5

1. As we mentioned earlier in 4.3, if we agree to have all sets of fuzzy sets of formulae at level-0 and hence collections like  $\{T_i\}_{i\in I},\ \{T_j\}_{j\in J},\ \dots$  then at level-1 there would be several notions of semantic consequence like  $\models_{`\{T_i\}'_{i\in I}}, \models_{`\{T_j\}'_{j\in J}}, \dots$  Now to talk about all these notions of semantic consequence we need to have the name of each such notions i.e. " $\models_{`\{T_i\}'_{i\in I}}$ ","  $\models_{`\{T_j\}'_{j\in J}}$ ",... and a variable  $\models_{\{T_v\}_{v\in V}}$  to range over all such  $\models_{`\{T_i\}'_{i\in I}}, \models_{`\{T_j\}'_{j\in J}}, \dots$  at level-2. With this language-background of level-2 we are able to write down the following level-2 assertions.

### Representation Theorem:

- (i) For any  $\models_{\{T_{\nu}\}_{\nu\in V}}$ ,  $\models_{\{T_{\nu}\}_{\nu\in V}}$  is a graded consequence relation.
- (ii) If " $\vdash$ " is a graded consequence relation, then there exists  $\models_{\{T_v\}_{v\in V}}$  such that  $\models_{\{T_v\}_{v\in V}}$  coincides with " $\vdash$ ".

These theorems are proved in [7]. Here are their formal presentations only in the current framework.

2. In reference to 3.8, if we admit to offer many-valued understanding to the notion of soundness/completeness too, then we can accommodate sentences like Sound ("  $\vdash$  ","  $\models_{\P T_i\}_{i\in I}}$ ") and Complete ("  $\vdash$  ","  $\models_{\P T_i\}_{i\in I}}$ ") by introducing Sound and Complete as two basic predicate symbols of level-2, and associate the values of the level-1 formulae, respectively  $\forall X \forall x ((X \vdash x) \rightarrow (X \models_{\P T_i})_{i\in I}^r x))$  and  $\forall X \forall x ((X \models_{\P T_i})_{i\in I}^r x) \rightarrow (X \vdash x))$  to the sentences.

#### 5. An analysis of existing fuzzy logics

5.1. The expressed meaning for consequence: Goguen to Haíek

Pavelka in [29] introduced the idea of fuzzy syntax on an abstract set F by incorporating two key concepts, namely, fuzzy set of axioms  $\mathscr A$  and fuzzy rule of inference. In designing a many-valued/fuzzy rule of inference, he took motivation from Goguen [17], in which the Modus Ponens rule intended to capture the following

"If you know P is true at least to the degree a and  $P \supset Q$  at least to the degree b then conclude that Q is true at least to the degree a.b"

['.' is the multiplication operator in the unit interval [ 0, 1 ]

An immediate formalized structure of the above idea of fuzzy rule of inference, is

$$(P, a)$$

$$(P \supset Q, b)$$

$$(Q, a.b)$$

That is, a rule of inference can be viewed as a subset of  $P(F \times L) \times (F \times L)$ .

[(L is considered as the truth set for the object level formu-

lae), which is a crisp relation and hence a crisp rule of inference.

Pavelka's own interpretation of a many-valued rule of inference is as follows: a many-valued rule of inference r consists of two components  $\langle r', r'' \rangle$ , where the first (grammatical) component r' operates on formulae and the second (evaluation) component r'' operates on truth values and says how the truth value of the conclusion is to be computed from the truth values of the premises.

From the above definition of many-valued rule of inference, a natural question emerges. Whether this many-valued rule of inference gives any grade to the rule (i.e. whether the rule is a fuzzy relation) or it simply is a crisp rule between a set of premises with given truth values, and a conclusion with a given truth value, the truth values of the premises as well as the conclusion being shown side by side? Pavelka, himself puts a many-valued rule of inference as,

$$\begin{pmatrix}
P & a \\
P \supset Q & b \\
\hline
Q & a*b
\end{pmatrix}$$

This is a replica of the form proposed by Goguen. ['\*' is the multiplication operator in a complete residuated lattice.]

The rules of fuzzy logic seem to be, in accordance with the words of Peter Hajek in [21], from partially true premises partially true conclusion can be deduced. Keeping this in mind Hajek proposed a fuzzy logical system, called Rational Pavelka Logic (RPL). Identifying the pair (P, a) with the formula  $\overline{a} \supset P$ , it can be shown, that the rule

$$\frac{(P, a)}{(P \supset Q, b)}$$
$$\frac{(Q, a.b)}{(Q, b)}$$

can be obtained as a derived rule in RPL, where  $\overline{a}$  is the wff denoting the truth value a. This again corroborates the idea, initiated by Goguen.

### 5.2. An analysis of existing fuzzy logics through distinction of levels

From the above discussion, it is quite visible that both Pavelka and Hajek had proposed a crisp relation in disguise of many-valued rule of inference. The following analysis will add some more strength to our observation.

(I) Let us first concentrate on Pavelka's approach. As previously mentioned in 5.1, it is quite transparent that Pavelka's notion of many-valued/fuzzy/approximate rule is simply a crisp relation between a set of partially true premises and a partially true conclusion. But to keep no room for doubt open we propose to analyze Pavelka's theory by assuming the notion of fuzzy rule as a fuzzy relation, in the level distinction framework. That is, instead of viewing the fuzzy rule Modus Ponens (see 5.1) as  $r(\{(P,a), (P \supset Q, b)\}, (Q, a*b))$ , let us consider r as a fuzzy relation  $r(\{(P,a), (P \supset Q, b)\}, Q)$  with the relatedness grade a\*b.

Now before entering into the rigorous process of level distinction let us first put a glance to the definition of proof, proposed by Pavelka.

**Pavelka's definition of proof**: Given a fuzzy subset  $\mathscr{A}$  of

formulae, interpreted as axioms and a set  $\mathscr{R}$  of rules of inference, an  $\mathscr{R}$ -proof is defined as a finite non-empty string, say  $\omega = \langle \omega_1, \omega_2, \ldots, \omega_n \rangle$  over  $F \cup (F \times \{0\}) \cup (F \times \mathscr{R} \times N^+)$ . That is for each  $\omega_i$   $(i=1,2,\ldots,n)$  either  $\omega_i$  is (x) or (x,0) or  $(x,r,\langle i_1,i_2,\ldots,i_n\rangle)$ , where  $x=\lceil \omega_i$ , the formula under consideration at the i-th term of  $\omega$ . If  $\omega_i = (x,r,\langle i_1,i_2,\ldots,i_n\rangle)$ ,  $(i=1,2,\ldots,n)$  then given r' as the grammatical component of r, x=r' ( $\lceil \omega_{i_1},\lceil \omega_{i_2},\ldots,\lceil \omega_{i_n}\rangle$ ). For an  $\mathscr{R}$ -proof  $\omega=\langle \omega_1,\omega_2,\ldots,\omega_n\rangle$  of  $\lceil \omega_n \rceil$  from a fuzzy subset of formulae X there is a function  $\widehat{\omega}: L^F \mapsto L$ , defined as follows.

(i) If length of  $\omega$  is 1, then either  $\omega = (x)$  or (x,0). Then the corresponding  $\widehat{\omega}(X) = Xx$  i.e. the membership degree of x in the fuzzy subset X when  $\omega = (x)$ , and  $\mathscr{A}x$  i.e. the membership degree of x in the fuzzy set of axioms  $\mathscr{A}$  when  $\omega = (x,0)$ .

(ii) If 
$$\omega = \langle \omega_1, \omega_2, \dots, \omega_n \rangle$$
 then 
$$\widehat{\omega}(X) = Xx \qquad \text{if } \omega_n = (x)$$
$$= \mathscr{A}x \qquad \text{if } \omega_n = (x,0)$$
$$= r''(\widehat{\omega}_{i_1}(X), \widehat{\omega}_{i_2}(X), \dots, \widehat{\omega}_{i_n}(X)) \qquad \text{if } \omega_n = (x,r,\langle i_1,i_2,\dots,i_n\rangle), \ i_1,i_2,\dots,i_n < n$$
 That is, if the target formula of some  $\omega_i$  in  $\omega$  is a consequence

That is, if the target formula of some  $\omega_i$  in  $\omega$  is a consequence of some rule r, applied on the formulae under consideration at  $\omega_{i_1}, \ \omega_{i_2}, \ \ldots, \ \omega_{i_n}, \ (i_1, i_2, \ldots, i_n < i)$  then the value of  $\langle \omega_1, \omega_2, \ldots, \omega_i \rangle$  depends on the value of  $\langle \omega_1, \omega_2, \ldots, \omega_{i_1} \rangle$ ,  $\langle \omega_1, \omega_2, \ldots, \omega_{i_2} \rangle, \ldots, \langle \omega_1, \omega_2, \ldots, \omega_{i_n} \rangle$ .

As an illustration, let us assume that in a proof  $\omega = \langle \omega_1, \omega_2, \ldots, \omega_n \rangle$  the fuzzy rule MP has to be applied on *i*-th and *j*-th steps (i, j < n) to get  $\lceil \omega_n$ . Then one should know the premises viz.,  $(\lceil \omega_i, \text{ the value of } \langle \omega_1, \omega_2, \ldots, \omega_i \rangle)$  and  $(\lceil \omega_j, \text{ the value of } \langle \omega_1, \omega_2, \ldots, \omega_j \rangle)$  before applying the rule as the application of the rule depends on both the formulae, and their values.

### Note 5.2.1

- (i) Use of r' and r'' in the above discussion denote the grammatical component and evaluation component of the rule r (see section 5.1) respectively.
- (ii) Pavelka has used the term 'Value of  $\omega$  (=  $\langle \omega_1, \omega_2, \ldots, \omega_n \rangle$ )'. One may think that it is the value of the proof  $\omega$  (=  $\langle \omega_1, \omega_2, \ldots, \omega_n \rangle$ ). But this is a value assigned at the end formula of the proof, computed via some method (see the sub-section 5.4). So, in our opinion, it should not be called the value of the proof as a whole.

### 5.3. Rewriting Pavleka's logic maintaining distinction of levels assuming r as a fuzzy relation

Level-0 consists of:

- All formulae  $\alpha$ ,  $\beta$ ,  $\gamma$ , ...
- All fuzzy sets of formulae  $\mathcal{A}$ , X, Y, Z, ...,  $T_1$ ,  $T_2$ ,  $T_3$ , .... ( $\mathcal{A}$  for axiom set, X, Y, Z etc for premises, and  $T_i$ 's for defining the semantics)
- All finite sequences of formulae  $\langle \alpha_1, \alpha_2, ..., \alpha_n \rangle$ ,  $\langle \beta_1, \beta_2, ..., \beta_m \rangle$ , ....
- All finite sequences over  $F \times L$  i.e.  $\langle (\alpha_1 a_1), (\alpha_2 a_2), \dots, (\alpha_n a_n) \rangle$ , ....
- A particular set r' containing all sequences of formulae of the form  $\langle \alpha, \alpha \supset \beta, \beta \rangle$  i.e.

$$r' = \{ \langle \alpha, \alpha \supset \beta, \beta \rangle, \langle \gamma, \gamma \supset \delta, \delta \rangle, \dots \}.$$

- All two length sequences over natural numbers i.e.  $\langle 1,2\rangle,$   $\langle 2,3\rangle,\ldots$ 

- All elements  $\omega_1,\,\omega_2,\,\ldots,\,\omega_n,\,\ldots$  where each  $\omega_i$  stands for an entity of the following kinds:

 $(\alpha)$ ,  $(\beta)$ ,  $(\gamma)$ , ....  $(\alpha, 0)$ ,  $(\beta, 0)$ ,  $(\gamma, 0)$ , ....  $(\alpha, r', \langle n_1, n_2 \rangle)$ ,  $(\beta, r', \langle s_1, s_2 \rangle)$ ,  $(\gamma, r', \langle m_1, m_2 \rangle)$ , ... where  $n_1$ ,  $n_2$ ,  $s_1$ ,  $s_2$ ,  $m_1$ ,  $m_2$  are finite natural numbers.

- (F) = {( $\alpha$ ), ( $\beta$ ), ( $\gamma$ ), ...}. - (F, 0) = {( $\alpha$ , 0), ( $\beta$ , 0), ( $\gamma$ , 0), ...}.

 $(x, \sigma) ((\alpha, \sigma), \langle \beta, \sigma \rangle, \langle \gamma, \sigma \rangle, \dots)$   $- (F \times r' \times N^+) = \{(\alpha, r', \langle n_1, n_2 \rangle), (\beta, r', \langle s_1, s_2 \rangle), (\gamma, r', \langle m_1, m_2 \rangle), \dots\}.$ 

- All finite sequences of the form  $\langle \omega_1, \omega_2, \dots, \omega_n \rangle$ , ....

- All finite sequences of the form  $\langle (\omega_1 \ a_1), \ (\omega_2 \ a_2), \dots, (\omega_n a_n) \rangle$ , ....

- All values  $a_1, a_2, ..., a_n, ...$ 

### Note 5.3.1

- 1. The fuzzy set of formulae  ${\mathscr A}$  is taken as a fuzzy set of axioms.
- 2. For convenience we have considered only the presence of a rule r' which actually represents the grammatical component of the rule Modus Ponens at the level-0 of Pavelka logic. In [29], mention of some rules other than Modus Ponens is also found, but for our present purpose taking just one rule Modus Ponens is sufficient.

### Level-1 consists of

- Names of each level-0 entity i.e. ' $\alpha', '\beta', '\gamma', \dots, '\alpha', 'X', 'Y', 'Z', \dots, '(\alpha_1, \alpha_2, \dots, \alpha_n)', '(\beta_1, \beta_2, \dots, \beta_m)', \dots, '(\alpha_1, \alpha_1), (\alpha_2, \alpha_2), \dots, (\alpha_n, \alpha_n) \rangle', \dots, 'r''$  ' $\omega_1', '\omega_2', \dots, '\omega_n', \dots, '(F)', '(F,0)', '(F\times r' \times N^+)', '(\omega_1, \omega_2, \dots, \omega_n)', \dots, '((\omega_1 \alpha_1), (\omega_2 \alpha_2), \dots, (\omega_n \alpha_n))', \dots '\alpha_1', '\alpha_2', \dots, '\alpha_n', \dots$  These are the constant symbols of the level-1 language.

- Variables: There are three types of variables one of each kind.

x (ranges over all formulae of level-0).

T (ranges over  $T_1, T_2, T_3, \dots$  of level-0).

 $\langle (), () \rangle$  (ranges over all 2-length sequences over  $F \times L$ .)

- Function symbol: [
- Predicate symbol:  $\in$ , r (both binary)
- Propositional connectives:  $\rightarrow_1$ ,  $\rightarrow_2$ , &,  $\lor$ .

(two implications, one conjunction and one disjunction)

- Quantifiers: ∀, ∃.
- Other symbols: ( , ), , .

Terms and well formed formulae are constructed as follows:

- Terms: (1) Constant symbols and variables are terms.

(2)  $\lceil \omega_1', \lceil \omega_2', \lceil \omega_3', \dots \text{ are terms.} \rceil$ (3)  $\lceil (\omega_1 a_1), (\omega_2 a_2), \dots, (\omega_n a_n) \rangle', \dots$ , etc are

(3) [' $\langle (\omega_1 \ a_1), (\omega_2 \ a_2), ..., (\omega_n \ a_n) \rangle$ ', ..., etc are also terms.

- Wffs: The formulae, below, represent each type of basic wffs of the lavel-1 language.

$$\begin{split} & `\alpha' \in `X', \, \ldots, \, `\alpha' \in T, \, \ldots, \, x \in `X', \, \ldots, \, x \in T, \, \ldots, \, `\alpha' \in `\mathscr{A}'. \\ & \lceil `\omega_1' \in `X', \, \ldots, \, \lceil `\omega_1' \in \mathscr{A}, \, `\omega_1' \in `(F)', \, \ldots, \, `\omega_1' \in `(F \times \{0\})', \end{split}$$

...,  $`\omega_1' \in '(F \times r' \times N^+)', ..., r('\langle (\alpha_1, a_1), (\alpha_2, a_2) \rangle', `\alpha_3'), ..., r(\langle (\alpha_1, a_1), (\alpha_2, a_2), (\alpha_2, a_2) \rangle', \alpha_3'), ..., r(\langle (\alpha_1, a_1), (\alpha_2, a_2), (\alpha_2$ 

Following expressions are examples of some compound wffs

formed by conjoining some of the above mentioned basic wffs by connectives.

'X'  $\subseteq$  'Y'  $\equiv$   $\forall$  x ( $x \in$  'X'  $\rightarrow_2 x \in$  'Y') [ read as 'X' is included in 'Y' ]

' $X' \models `\alpha' \equiv \forall \ T \ (X' \subseteq T \rightarrow_1 `\alpha' \in T)$ . [ read as '\alpha' is a semantic consequence of 'X']

Taut(' $\alpha$ ')  $\equiv \forall T$  (' $\alpha$ '  $\in T$ ) [ read as ' $\alpha$ ' is a tautology ]

### 5.4. Difficulty in writing the formal definition of proof of Pavelka in this context

The notion of proof (in fact a derivation which Pavelka termed as proof) is to be defined by a level-1 sentence. If this sentence is many-valued, proof is also a many-valued concept. Now following Pavelka we can write a sequence ' $\langle \omega_1, \omega_2, \ldots, \omega_n \rangle$ ' as a proof of  $\lceil \omega_n \rceil$ ' from a premise 'X' if and only if

[ (if ' $\omega_1$ 'is in '(F)' then [' $\omega_1$ ' is in 'X') and (if ' $\omega_1$ ' is in '(F×{0})' then [' $\omega_1$ ' is an axiom) ] and [ (if ' $\omega_2$ ' is in '(F)' then [' $\omega_2$ ' is in 'X') and (if ' $\omega_2$ ' is in '(F×{0})' then [' $\omega_2$ ' is an axiom) ] and [ (if ' $\omega_3$ ' is in '(F)' then [' $\omega_3$ ' is in 'X') and (if ' $\omega_3$ ' is in '(F×{0})' then [' $\omega_3$ ' is an axiom) and (if ' $\omega_3$ ' is in '(F× $r' \times N^+$ )' then [' $\omega_3$ ' is a result of a rule r) ] and ... and [ (if ' $\omega_n$ ' is in '(F)' then [' $\omega_n$ ' is in 'X') and (if ' $\omega_n$ ' is in '(F× $\ell$ )' then [' $\omega_n$ ' is an axiom) and (if ' $\omega_n$ ' is in '(F× $\ell$ )' then [' $\omega_n$ ' is a result of a rule r) ].

#### Note 5.4.1

As mentioned earlier, for simplicity we have taken only one rule r to analyze Pavelka's logic. Instead of one rule if a set of rules R consisting of some finite number of rules, say  $r_1$ ,  $r_2$ , ...,  $r_n$  is taken, then for each  $\omega_i$   $(1 < i \le n)$ , the last conjunct of each  $\int$  will be (if ' $\omega_i$ ' is in ' $(F \times R \times N^+)$ ' then ( $\lceil `\omega_i `$  is a result of a rule  $r_1$  or  $\lceil `\omega_i `$  is a result of a rule  $r_2$  or ... or  $\lceil `\omega_i `$  is a result of a rule  $r_n$ )). In this case the level-1 language needs to have the connective  $\vee$ .

In this context, where only the presence of rule r has been assumed, the above defining condition turns out to be the following wff of level-1.

$$\begin{bmatrix} (`\omega_1` \in `(F)` \to_1 \lceil `\omega_1` \in `X') \& (`\omega_1` \in `(F \times \{0\})` \to_1 \lceil `\omega_1` \\ \in \mathscr{A}) \end{bmatrix} \&$$

$$\begin{bmatrix} (`\omega_2` \in `(F)` \to_1 \lceil `\omega_2` \in `X') \& (`\omega_2` \in `(F \times \{0\})` \to_1 \lceil `\omega_2` \\ \in \mathscr{A}) \end{bmatrix} \&$$

$$\begin{bmatrix} (`\omega_3` \in `(F)` \to_1 \lceil `\omega_3` \in `X') \& (`\omega_3` \in `(F \times \{0\})` \to_1 \lceil `\omega_3` \\ \in \mathscr{A}) \& \\ (`\omega_3` \in `(F \times r' \times N^+)` \to_1 Res_r(\lceil `\omega_3`)) \end{bmatrix} \& \dots \& \begin{bmatrix} (`\omega_n` \in `(F)` \to_1 \lceil `\omega_n` \in `X') \& \\ (`\omega_n` \in `(F \times \{0\})` \to_1 \lceil `\omega_n` \in \mathscr{A}) \& (`\omega_n` \in `(F \times r' \times N^+)` \to_1 Res_r(\lceil `\omega_n`)) \end{bmatrix} . \dots (1)$$

 $Res_r[`\omega_i']$ , defined formally below, stands for  $[`\omega_i']$  is a result of some rule r,  $(2 < i \le n)$ . So, the value of the sentence (1) is to be calculated by considering values of all its conjuncts. But Pavelka has considered only the last component of the expression (1), and i.e. the sentence under the last tagged the value of it with the sentence ' $\langle \omega_1, \omega_2, ..., \omega_n \rangle$ ' is a proof of  $\lceil \omega_n \rceil$  from 'X'. This is the first difficulty.

There is a second difficulty whatever value may be taken. We consider the value taken by Pavelka viz. the value of the last conjunct in (1).

Let us now, concentrate on the part  $Res_r(\lceil \omega_i')$ .  $Res_r(\lceil \omega_i')$  is actually the abbreviation of  $\exists \langle (), () \rangle (\langle (), () \rangle \subseteq [ ((\omega_1 a_1), (\omega_1 a_1), (\omega_1$  $(\omega_2 a_2), \ldots, (\omega_{i-1} a_{i-1})$   $\rangle$   $\& r(\langle (), () \rangle, [\omega_i])$  which can be read as, there is  $\langle (\ ), (\ ) \rangle$  such that  $\langle (\ ), (\ ) \rangle$  is included in [' $\langle (\omega_1 a_1), (\omega_2 a_2), ..., (\omega_{i-1} a_{i-1})$ ' and  $\langle (), () \rangle$  is related with  $\lceil \omega_i \rceil$  by the rule r.

Here  $a_k$ , k = 1, 2, ..., i-1, refers to the truth value of the level-1 sentence

$$\begin{bmatrix} (\omega_k' \in (F)' \to_1 \lceil \omega_k' \in X') & (\omega_k' \in (F \times \{0\})' \to_1 \lceil \omega_k' \\ \in \mathscr{A}) & \end{bmatrix}$$

 $('\omega_k' \in '(F \times r' \times N^+)' \rightarrow_1 Res_r(\lceil '\omega_k'))$  that is the last conjuncts of  $\langle \omega_1, \omega_2, \dots, \omega_k \rangle$  is a proof of  $\vec{\Gamma} \omega_k$  from a premise

Now, we arrive at the most crucial formal difficulty. Attention of readers is being drawn to the point that  $a_k$  is the name of the value of the level-1 sentence

$$\begin{bmatrix} (\omega_k' \in (F)' \to_1 [\omega_k' \in X') & (\omega_k' \in (F \times \{0\})' \to_1 [\omega_k' \in A'] & (\omega_k' \in (F \times \{0\})' \to_1 [\omega_k' \in A'] & (\omega_k' \in A'] & (\omega_k' \in A') & (\omega_k'$$

 $(\omega_k' \in (F \times r' \times N^+)' \to_1 Res_r([\omega_k'))$  which is being used at the same level. The name of the value of a level-1 sentence should be an entity of level-2, but here that has been used at level-1. But as a name it is also mentioned at

So, the main question is about the well-formedness of the wff  $Res_r([\omega_i])$ .

**Example** We will now consider one example explaining both the difficulties mentioned above. Let *X* be the fuzzy subset such that  $X(\alpha) = .7, X(\alpha \supset \beta) = .3, X(\beta \supset \gamma) = .9$  and 0 for the rest of the formulae. Let us consider the proof  $\omega$  of  $\gamma$  from X as proposed by Pavelka. Let  $\omega = \langle \omega_1, \omega_2, \omega_3, \omega_4, \omega_5 \rangle$ , where  $\omega_1 = (\alpha)$ ,  $\omega_2 = (\alpha \supset \beta)$ ,  $\omega_3 = (\beta, r, \langle 1, 2 \rangle)$ ,  $\omega_4 =$  $(\beta \supset \gamma)$ ,  $\omega_5 = (\gamma, r, \langle 3, 4 \rangle)$ . That is, justification of the steps  $\omega_1, \omega_2$ , and  $\omega_4$  is that they are taken from premise, and for the steps  $\omega_3, \omega_5$  the justification is that they are results of the rule *r* applied on steps 1, 2 and 3, 4 respectively.

Hence 
$$\langle \omega_1 \rangle(X) = .7$$
,  $\langle \omega_1, \omega_2 \rangle(X) = .3$ ,  $\langle \omega_1, \omega_2, \omega_3 \rangle(X) = .7 * .3$ ,  $\langle \omega_1, \omega_2, \omega_3, \omega_4 \rangle(X) = .9$ ,  $\langle \omega_1, \omega_2, \omega_3, \omega_4, \omega_5 \rangle(X) = .7 * .3 * .9$ .

(i) As given by the expression (1) of the previous

 $(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5)$  is a proof of  $\gamma$  from X, if we consider value of each step of the proof  $\langle \omega_1, \omega_2, \omega_3, \omega_4, \omega_5 \rangle$  i.e. the value of each conjunct separated by , then the value should be .7 \* .3 \* (.7 \* .3) \* .9 \* (.7 \* .3 \* .9); but instead of that Pavelka considered the value of the expression bracketed by the last of the entire expression for proof presented in (1). So, following Pavelka .7 \* .3 \* .9 supposed to be attached as the value of the proof of  $\gamma$  from X.

(ii) As proposed by Pavelka, the value of  $\langle \omega_1, \omega_2, \omega_3 \rangle(X)$ 

$$\begin{bmatrix} ('\omega_3' \in '(F)' \rightarrow_1 \lceil '\omega_3' \in 'X') & ('\omega_3' \in '(F \times \{0\})' \rightarrow_1 \lceil '\omega_3' \in \mathscr{A}) & \\ ('\omega_3' \in '(F \times r' \times N^+)' \rightarrow_1 Res_r(\lceil '\omega_3')) \end{bmatrix}.$$

But as  $\omega_3 = (\beta, r, \langle 1, 2 \rangle) \in (F \times r' \times N^+)$ , the value of the first two conjunct of the above expression would be 1. Hence the value of the whole expression would be value( $Res_r(\lceil \omega_3')$ ). Now  $Res_r(\lceil \omega_3') \equiv$ 

$$\begin{split} &\exists \ \langle (\ ), (\ ) \rangle \left( \langle (\ ), (\ ) \rangle \subseteq \lceil \langle \ (\omega_1 \ \overline{\langle \omega_1 \rangle}(X)), \ (\omega_2 \ \overline{\langle \omega_1, \omega_2 \rangle}(X)) \ \rangle \rangle \right. \\ &\otimes r(\langle (\ ), (\ ) \rangle \ , \lceil \langle \omega_3 \rangle) \right) \\ &\equiv \exists \ \langle (\ ), (\ ) \rangle \left( \langle (\ ), (\ ) \rangle \subseteq \lceil \langle \ (\omega_1, \, `.7'), \ (\omega_2, \, `.3') \ \rangle \ \& \ r(\langle (\ ), (\ ) \rangle \right. \\ &\wedge , \lceil \langle \omega_3 \rangle \right). \end{split}$$

So, we need to refer to  $(\omega_1)(X)$ , the value of  $(\omega_1)$  and  $\langle \omega_1, \omega_2 \rangle (X)$ , the value of  $\langle \omega_1, \omega_2 \rangle$  at level-1 where the value of the proof itself is an entity. This issue will be discussed more elaborately in Note 5.4.3.

#### Note 5.4.2

It may be noted that two different implications are required in formalizing Pavelka logic, one for defining the notion of inclusion (⊆), and the other for the notion of semantic consequence. The detail is given below in the interpretation of level-1 language. This point however is not explicitly mentioned in [20].

Interpretation of level-1 language:

- 1. Interpretation of constant symbol is the corresponding level-0 entity.
- 2. ' $\omega_i$ ' refers to any one of  $(\alpha)$ ,  $(\alpha, 0)$ , and  $(\alpha, r', \langle n_1, n_2 \rangle)$ .
- 3. [' $\omega_i$ ' refers to the formula  $\alpha$  when  $\omega_i = (\alpha)$  or  $\omega_i = (\alpha, 0)$ or  $\omega_i = (\alpha, \mathbf{r}', \langle n_1, n_2 \rangle)$  for some natural numbers  $n_1, n_2$ .
- 4.  $\lceil \langle (\omega_1 \ a_1), \ (\omega_2 \ a_2), \ \dots, \ (\omega_n \ a_n) \rceil \rceil$  refers to  $\langle (\text{the first}) \rceil$ component of the referent of ' $\omega_1$ ',  $a_1$ ), (the first component of the referent of ' $\omega_2$ ',  $\alpha_2$ ), ..., (the first component of the referent of ' $\omega_n$ ',  $a_n$ ).
- 5. Interpretation of the predicate r is given by the fuzzy relation,

Int(r): (set of all 2-length sequence over  $F \times L$ )  $\times F$ :  $\mapsto L$ (L is the truth set for the level-1 language), defined by, Int $(r)(\langle (\alpha_1 \ a_1), (\alpha_2 \ a_2) \rangle, \alpha_3) = r''(a_1, a_2), \text{ if } \langle \alpha_1, \alpha_2, \alpha_3 \rangle \text{ is}$ in r'.

$$= 0$$
, otherwise.

[ r', r'' are as mentioned in Pavelka [20] ]

5. Value assigned to a level-1 formula of the form ' $\alpha$ '  $\in$  'X' page, in order to calculate the value of the sentence is the value assigned to  $\alpha$  by the fuzzy subset, referred to by 'Χ'.

- 6. From [29], some tacit references suggest to consider a complete lattice as the truth structure for the metalanguage of Pavelka logic. So, within this structure one can define an operator  $\Rightarrow_1$  by,  $a \Rightarrow_1 b = \sup\{z : a \land z \le b\}$  to interpret the level-1 connective  $\rightarrow_1$ .
- 7. In accordance with the definition of inclusion relation between two fuzzy subsets, proposed by Pavelka [29], no grade other than 0 and 1 is intended to be attached to the sentence

'X'⊆ 'Y'. So, interpretation of  $\rightarrow_2$ , denoted by  $\Rightarrow_2$  should be given by,

$$a \Rightarrow_2 b = 1,$$
  $a \le b.$   
= 0, otherwise.

8. Interpretation of & and  $\lor$  are given by lattice meet and join respectively.

#### Note 5.4.3:

1. According to the methodology, each level consists of two parts, namely, a language and its corresponding algebra. To talk about a sentence or value of a sentence, present at some level, one has to go to the next higher level. That is an entity of a level cannot be both used and mentioned at the same level. But, in designing  $Res_r([\omega_i])$  the above mentioned principle has to be violated if one desires to accommodate Pavelka's notion of proof with the assumption that ris a fuzzy rule (i.e. a fuzzy relation). Proof is a notion that has been 'used' at level-1 of Pavelka logic, and so value of a proof cannot be referred to (see 5.2.2) i.e. 'mentioned' at that level. Besides this, while writing a proof, as the value of a sub-proof is being incorporated within the body of the rule, it is also 'used' at the same level. So, maintaining of level distinction properly does not allow us to see the metalogical notion, namely, ' $\alpha$ ' is a syntactic consequence of 'X' as a many-valued notion.

That is, we can say that  $(\beta, a * b)$  follows from  $(\alpha, a)$  and  $(\alpha \supset \beta, b)$ . But if we want to say that from  $(\alpha, a)$  and  $(\alpha \supset \beta, b)$ ,  $\beta$  follows, and the truth of this following is to the extent a \* b then we would make a mistake in the categories of 'using' and 'mentioning'.

- 2. Hajek [21] has presented a comparatively simpler version of Pavelka logic and named it as Rational Pavelka Logic. We have tried to analyze Hajek's presentation (see section 5.5) from the same angle and faced a similar problem there too.
- 3. Moreover in [29], assumptions regarding metalevel truth structure had not been made explicit. But some hints of thoughts, proposing a complete lattice to be a structure for metalevel language, can be found [29]. But to see the value attached with the notion of semantic consequence in Pavelka's case as a value of the *sentence* determining *the notion of semantic consequence* we need to have two implications as mentioned above. In this regard the level-1 connective  $\rightarrow_1$  needs to be interpreted as  $a \Rightarrow_1 b = \sup\{z: a \land z \leq b\}$ , which is available in a complete lattice. But there is no specification about the operator, interpreting the connective  $\rightarrow_2$  in [29]. Without this implication the subset relation of fuzzy sets cannot be defined. So, as the notion of semantic consequence i.e. 'X' |= '\alpha' is to be defined by both the connectives  $\rightarrow_1$ ,  $\rightarrow_2$ , (i.e. two implications are needed

at the same level) one can be critical about the notion of semantic consequence itself.

### 5.5. A brief revisit to Hajek's logic RPL

After analyzing Pavelka's approach let us explore Hajek's logic RPL. As found in [21], there is a notion viz. the 'provability degree of  $\alpha$  from X', defined by,

Provability degree of  $\alpha$  from  $X = \sup \{a : X \vdash (\alpha, a) \}$ 

Hajek presented RPL as an extension of Łukasiewicz logic. Language of RPL is extended by the names of the rational truth values in [0, 1] and then two axioms viz.,

 $\overline{a} *_{\mathbf{L}} \overline{b} \equiv \overline{a} \&_{\mathbf{L}} \overline{b}, \overline{a} \Rightarrow_{\mathbf{L}} \overline{b} \equiv \overline{a} \supset_{\mathbf{L}} \overline{b}$  (where  $*_{\mathbf{L}}$  and  $\Rightarrow_{\mathbf{L}}$  are Łukasiewicz's t-norm and its corresponding residuum) are added with the axioms of Łukasiewicz logic. Hajek used  $(\alpha, a)$  as an abbreviation for the formula  $\overline{a} \supset_{\mathbf{L}} \alpha$ , and with the above mentioned set of axioms and rule modus ponens he proved that the following rule can be derived in RPL.

$$\frac{\overline{a} \supset_{L} \alpha}{\overline{b} \supset_{L} (\alpha \supset_{L} \beta)}$$
$$\overline{a *_{L} \overline{b} \supset_{L} \beta}$$

This is same as Pavelka's many-valued rule MP. Now in RPL,  $X \vdash (\alpha, a)$  i.e.  $X \vdash \overline{a} \supset_{\overline{L}} \alpha$  is a wff of level-1. But as discussed earlier, from a crisp set X of formulae, following a crisp set of axioms and crisp rules of inferences (see section 5.1) a formula of the form  $(\alpha, a)$  can be derived or cannot be derived i.e. the notion of derivation is two-valued. The definition of 'provability degree of  $\alpha$  from X' suggests to compute the supremum of all those a for which  $(\alpha, a)$  is derived from X. This leads to a number of problems.

- 1. ' $(\alpha, a)$  is derived from X' is a wff of level-1. To compute the 'provability degree of  $\alpha$  from X' one needs to extract out those values a for which  $(\alpha, a)$  i.e.  $\overline{a} \supset_{\mathbb{L}} \alpha$  is derivable from X and put them together for computing the supremum. But the question is, where at which level this process of extracting out of the value a is taking place? Provability is a notion of level-1. So, keeping that in mind, if we assume that extracting out a from  $X \vdash \overline{a} \supset_{\mathbb{L}} \alpha$  is a level-1 activity then dequoting  $\overline{a}$  means using the value a which is referred to by  $\overline{a}$  at the same level, takes place. So, computing 'sup' over all those a for which  $X \vdash \overline{a} \supset_{\mathbb{L}} \alpha$  holds, is an external device, which is difficult to place suitably at any level.
- 2. It should be natural to think that the provability degree of a formula  $\alpha$  from a set of formulae X would be the truth value of the statement ' $\alpha$  is provable from X'. The value should not be a mere number, counted by some external device, depending on the notion ' $(\alpha, a)$  is derived from X'.
- 3. Ususally, ' $\alpha$ ' is provable from 'X' should be understood by the sentence viz., there is a derivation of ' $\alpha$ ' from 'X' or some variant form of it. 'sup' is usually used as an operator for existential quantifier 'there is'. But the way 'sup' is used in the definition of provability degree shows that neither it is used to compute the truth value of the sentence there is a derivation of ' $\alpha$ ' from 'X' nor for computing the truth value of the sentence there is a derivation of '( $\alpha$ ,  $\alpha$ )' from 'X'.

So, Hajek's proposed RPL also cannot address many-valuedness at metalogical concepts genuinely.

### 6. Comparison of the theory of graded consequence and other existing fuzzy logics

In the preceding sections, a somewhat detailed analysis of the theory of graded consequence and existing fuzzy logics is made. In the light of that as well as some other results obtained in earlier works we present the following observations.

1. In the construction of a fuzzy logic, both Pavelka and Hajek did not intend to mean a rule, a special case of a derivation, as a fuzzy relation. Rather they proposed an approximate rule to be an ordinary relation between a partially true premises (i.e. a set of formulae with many-valued truth values) and a partially true conclusion, a formula having truth value not necessarily true or false. So it seems natural to interpret that, for a fuzzy set A of axioms and many-valued rules  $\mathcal{R}$ ,  $C_{\mathcal{A},\mathcal{R}}(X)(\alpha)$  would be the value assigned to  $\alpha$  through the derivation process, not the value to the derivation process itself.

Whereas in the theory of graded consequence both the notions of syntactic and semantic consequence are meant to be a fuzzy relation i.e. the sentence ' $\alpha$  is a syntactic/semantic consequence of X' may get a value other than 0 and 1.

2. In classical two-valued situation, both the notions of consistency (inconsistency) and consequence are related by some transformations i.e. one can be obtained considering the other as a primitive notion. According to [29], a fuzzy set X of formulae is said to be consistent if  $C(X) \neq 1$  i.e. C(X) is not the whole set of formulae. Thus, according to Pavelka's definition, consistency is a crisp notion. So, the question arises, how could Pavelka make the notion of consistency, a two-valued notion and the notion of consequence, if grade is being attached to it, commensurate with each other in a sense similar to that in classical logic?

But in the theory of graded consequence, both the concepts have been developed, maintaining parity with classical case. Theory of graded consequence proposes notion of inconsistency as a many-valued notion, postulated as follows. For any set X of formulae, INCONS(X) is interpreted as 'the degree to which X is inconsistent', and the fuzzy subset INCONS over the set of formulae obeys the following axioms.

- 1. If  $X \subseteq Y$  then INCONS(X)  $\leq$  INCONS(Y).
- 2.  $INCONS(X \cup Y) \land INCONS(X \cup {\neg y}) \le INCONS(X)$  for any  $y \in Y$ .
- 3. There is some k > 0 such that  $\inf_{\alpha} INCONS(\{\alpha, \neg \alpha\}) = k$ .

Parallel to these axioms for graded inconsistency, notion of graded consequence (see Section 2) has been extended [8, 9] assuming the presence of negation  $(\neg)$  in the object language, and the new set of axioms is as follows.

- 1. If  $\alpha \in X$  then  $gr(X \mid \sim \alpha) = 1$
- 2. If  $X \subseteq Y$  then  $gr(X \mid \sim \alpha) \le gr(Y \mid \sim \alpha)$
- 3.  $inf_{\beta \in \mathbb{Z}} \operatorname{gr}(X \mid \sim \beta) \wedge \operatorname{gr}(X \cup Y \mid \sim \alpha) \leq \operatorname{gr}(X \mid \sim \alpha)$
- 4. There is some k > 0 such that,  $\inf_{\alpha,\beta} \operatorname{gr}(\{\alpha, \neg \alpha\})| \sim \beta = k$
- 5.  $\operatorname{gr}(X \cup \{\alpha\}) \mid \sim \beta \wedge \operatorname{gr}(X \cup \{\neg \alpha\}) \mid \sim \beta \leq \operatorname{gr}(X \mid \sim \beta)$ .

As in classical case, in the theory of graded consequence too both these generalized notions of consequence and inconsistency i.e. graded consequence and graded inconsistency are equivalent [8, 9]. 3. In the graded consequence approach the starting point constitutes of a set  $\{T_i\}_{i\in I}$  of fuzzy subsets of the set of formulae. This set may be interpreted as the context, the states of affair, the semantic base, or a set of valuations relevant for the logic. The logic which is determined by its consequence relations, both semantic and syntactic, depends on it. Depending upon the users' purpose the relevant fuzzy subsets  $T_i$  are determined. This part is, however, not the logic activity proper. The user has the freedom of choosing the logic depending on the purpose.

#### 7. Concluding remarks

One may wonder why all these tedious exercise of discerning levels by using various signs are presented at all. After all no new logic has been proposed, no new theorem has been proved. In summary here is the answer. Zadeh's own efforts of fuzzy logic centres around development of calculi - calculi of obtaining by mathematical operations on given fuzzy sets present in the premises a fuzzy set as the conclusion. Peter Hajek claims that Zadeh's method may be accommodated within the framework of fuzzy logic that he (Hajek) proposed in traditional syntactico-semantic style. Ever since the seminal work of Goguen, many researchers e.g. Pavelka [29], Hajek [21], Gottwald [18], Novak [27, 28], Mundici, Cignoli, D'Ottavino [24], Di. Nola, Sessa and Pedrycz [25, 26], Gerla and Biaccino [1, 2, 3, 4], Godo and Esteva [20] have dealt with various types of fuzzy logic. In a recent publication Godo, Esteva and Noguera have dealt with three types of soundness and completeness results for 'core predicate fuzzy logic' in the context of continuous t-norms [14]. But none, perhaps, has shown any inclination towards fuzziness of the metalevel concepts except Gerla's [16] marginal involvement in the issue. Pavelka's approach apparently shows that the concept of logical consequence has been made fuzzy but that is not correct. It should be re-emphasized that his intention was not to address this issue either. On the other hand, from the very first paper of Chakraborty [6], the attempt was precisely to introduce vagueness or more accurately many-valuedness at the metalevel notions. The necessity for, and justification of such an attempt are discussed in many subsequent papers [5, 7, 8, 9, 13] as mentioned in the introducion. That fuzzy logic needs to incorporate multivalence in its metalogic too also was felt by Parikh [30] in a somewhat different context of dealing with vagueness. It is also mentioned that Pelta has recently raised this issue [31]. In one of the papers of Gottwald [19] an idea for degree of soundness of a rule is mentioned. An comparison of this notion with our notion of graded consequence has been made in [10]. Besides, in a recent paper [37], Zadeh has introduced an idea viz. extended fuzzy logic which is intended to be an addition to fuzzy logic in the sense that it deals with the concept of fuzzily-valid (f-valid) reasoning. This point of view appears close to the notion of graded consequence. But in this study the conclusion, that approaches to, is fuzzy logics made so far other than the graded consequence do not address vagueness of the metalogical notions genuinely.

### References

- [1] Biacino, L., Gerla, G., Closure systems and L-subalgebras, Inform. Sci., 32, 181-195, 1984.
- [2] Biacino, L., Gerla, G., Closure operators for fuzzy subsets, EUFIT 93, First European Congress on Fuzzy and Intelligent Technologies, Aachen, 1441-1447, 1993.
- [3] Biacino, L., Gerla, G., An extension principle for closure operators, J. of Math. Anal. Appl., 198, 1-24, 1996.
- [4] Biacino, L., Gerla, G., Closure operators in fuzzy set theory, in fuzzy sets, in approximate reasoning and information systems, Edited by J.C.Bezdek, D. Dubois, H. Prade, Kluwer Academic Publishers, Dordrecht, 243-278, 1999.
- [5] Basu, S., A Study in Logical and Philosophical Implications of Graded Consequence in Many-valued Systems, a Ph.D thesis, Jadavpur University, 2003.
- [6] Chakraborty, M.K., Use of fuzzy set theory in introducing graded consequence in multiple valued logic, in M.M. Gupta and T. Yamakawa (eds.), Fuzzy Logic in Knowledge-Based Systems, Decision and Control, Elsevier Science Publishers, B.V.(North Holland) 247-257, 1988.
- [7] Chakraborty, M.K., Graded Consequence: further studies, Journal of Applied Non-Classical Logics, 5(2), 127-137, 1995.
- [8] Chakraborty, M.K., and Basu, S., Introducing grades to some metalogical notions, Proc. Fuzzy Logic Laboratorium, Linz, 85-100, 1996.
- [9] Chakraborty, M.K., and Basu, S., Graded Consequence and some Metalogical Notions Generalized, Fundamenta Informaticae 32, 299-311, 1997.
- [10] Chakraborty, M.K., and Dutta, S., Graded Consequence Revisited, Fuzzy Sets and Systems, 161, 1885-1905, 2010.
- [11] Church, A., Introduction to Mathematical Logic, vol.1, Princeton University Press, 58-61, 1964.
- [12] Dubois, D., Lang, J., Prade, J., Possibilkistic Logic, In handbook of Logic in Artificial Intelligence and Logic Programming, Vol-3, D.M.Gabbay and C.J. Hogger, eds. 439-510, Clarendon Press, Oxford, 1994.
- [13] Dutta, S., Basu, S. and Chakraborty, M.K., Many-Valued Logics, Fuzzy Logics and Graded Consequence: A Comparative Appraisal, K. Lodaya (ed.): ICLA 2013, LNCS 7750, Springer-Verlag Berlin Heidelberg, pp. 197-209, 2013.
- [14] Esteva, E., Godo, L., Noguera, C., On Completeness results for Predicate Łukasiewicz, Product, Gödel and Nilpotent Minimum Logics expanded with truth constants, Mathware and Soft Computing, Vol. XIV, n.3, 233-246, 2007.

- [15] Gentzen, G., Investigations into Logical Deductions, in the collected papers of G. Gentzen, M.E. Szabo, ed., 68-131, North Holland Publications, Amsterdam, 1969.
- [16] Gerla, G., Fuzzy Logic: Mathematical Tools for Approximate Reasoning, Kluwer Academic Publishers, 2001.
- [17] Goguen, J.A., The logic of inexact concept, Synthese, 19, 325-373, 1968.
- [18] Gottwald, S., Fuzzy sets and fuzzy logic, Vieweg, Wiesbaden, 1993.
- [19] Gottwald, S., An approach to handle partially sound rules of inference, In: Advances in Intelligent Computing, IPMU'94, Selected papers, B. Bouchon-Meunier, R. R. Yager, L. A. Zadeh (eds.), Lecture Notes Computer Sci., Vol-945, Springer: Berlin etc., 380 - 388, 1995.
- [20] Hajek, P., Godo, L., Esteva, F., A complete many-valued logic with product Conjunction, Arch. Math Logic, Kluwer Academic Publishers, 35, 191-208, 1996.
- [21] Hajek, P., Metamathematics of Fuzzy Logic, Kluwer Academic Publishers, 1998.
- [22] Hunter, G., Metalogic: An Introduction to the Meta Theory of Standard First Order Logic, Macmillan Students' edition, 1969.
- [23] Kleene, S. C., Introduction to Metamathematics, 7th edition, Walterf-Noorbhoff publishing, Groningen and North-Holland publishing company, Amsterdam, 1921.
- [24] Mundici, D., Cignoli, R., D'Ottavino, I.M.L., Algebraic Foundations of Many- valued Reasoning, Trends in Logic, Vol. 7, Kluwer Academic Publishers, Dordrecht, 2000.
- [25] Nola, Di, MV-Algebras in the treatment of uncertainty, in proceedings of IFSA Congress, Lowen, Roubens eds., Kluwer, Dordrecht, 123-131, 1993.
- [26] Nola, Di, Sessa, S., Pedrycz, W., Modus Ponens for fuzzy data realized via. Equalitions with equality operators, Int. J. of intelligent Systems, 5, 1-14, 1990.
- [27] Novak, V., On syntactico-semantical completeness of first order fuzzy logic, Parts I And II, Kybernetica, 2, 6(1,2), Academia Praha, 47-154, 1990.
- [28] Novak, V., Perfilieva, I., Mockor, J., Mathematical Principles for Fuzzy Logic, Kluwer, 1999.
- [29] Pavelka, J., On fuzzy logic I, II, III, Zeitscher for Math. Logik und Grundlagen d. Math 25, 45-52, 119-134, 447-464, 1979.
- [30] Parikh, R., The Problem of Vague Predicates. In: Cohen, R.S., Wartofsky, M. (eds.) Language, Logic, and Method, 241âĂŞ261. D. Ridel Publishing Company, 1983.
- [31] Pelta, C., Wide sets, Deep many-valuedness and Sorites arguments, Mathware and Soft computing, 11, 5-11, 2004.
- [32] Shoesmith, D.J. and Smiley, T.J., Multiple Conclusion Logic, Cambridge University Press, 1978.

- [33] Tarski, A., Methodology of Deductive Sciences, In Logic, Semantics, Metamathematics, 60-109, Clavendon Press, 1956.
- [34] Zadeh, L.A., Fuzzy sets, Information and Control, 8, 338-353, 1965.
- [35] Zadeh, L.A., PRUF- a meaning representation language for natural languages, Int. J. Man-Machine Studies, 10,
- 395-460, 1978.
- [36] Zadeh, L.A., Generalized Theory of Uncertainty (GTU)principal concepts and ideas, Science Direct, Elsevier, Computational Statistics and Data Analysis, 51, 15-46, 2006.
- [37] Zadeh, L.A., Toward Extended Fuzzy Logic A First Step, Fuzzy Sets and Systems, 160, 3175 3181, 2009.

## Comparing Vague Preferences in Recommender Systems

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The main goal of a recommender system is to generate meaningful recommendations to a group of users for items or products that might interest them. Two basic architectures are usually applied in recommender systems: content-based filtering (focused on the similarity of items determined by measuring the similarity in their properties) and collaborative filtering systems (focused on the similarity of items determined by the similarity of the ratings of those items rated by the users).

Measuring similarity between preferences of the users is a crucial point in the collaborative filtering. Actually, a recommender system suggests to a user available resources using similarity of his/her preferences to those of the other users. These preferences may be represented by an ordering set by a user of the items he/she have seen earlier. If an ordering of a user A is similar to an ordering of another user B then the recommender system tends to suggest to user A another resources highly preferred by B and not yet seen by A.

Preferences for certain items in a recommender system are stored in the so-called utility matrix with rows corresponding to the users, columns representing items, and the entries showing ratings attributed by each user to particular items. Unfortunately, real utility matrices are sparse since many entries are unknown. Moreover, another drawback that appears often when we examine humans opinions expressed by rankings are ties, i.e. cases when the same rank is assigned to more than one element. Such ill-defined preferences including gaps or ties (further on called vague preferences) require special tools for measuring similarities.

In this paper we propose a novel approach for measuring similarity between preferences modeled by the intuitionistic fuzzy sets [1] (later on called IF-sets). As it was shown in [2, 3, 4] IF-sets may appear very useful in modeling vague preferences admitting ties and lack of information.

The starting point of our considerations was to establish some natural and intuitive requirements that each reasonable similarity measure for comparing two (vague) preference systems should possess. Next we have examined a broad collection of well-known IF-distances, IF-dissimilarities and IF-divergences (see [6, 7]) which might be perceived as possible candidates for measuring similarity between preferences.

It turned out that many of those functions, although useful in many other situations, were not suitable for our goal. Finally, two IF-distances (i.e. the Euclidean-type IF-distance given by Atanassov [1] and the distance suggested by Hong and Kim [5]), that satisfy all desired conditions, were chosen. Then two similarity measures, based on the above mentioned IF-distances, that might be applied in recommender systems were proposed and their properties were examined.

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### References

- [1] Atanassov K., Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20, pp. 87-96 (1986).
- [2] Grzegorzewski P, The coefficient of concordance for vague data, *Computational Statistics & Data Analysis*, 51, pp. 314-322 (2006).
- [3] Grzegorzewski P., KendallÓş correlation coefficient for vague preferences, *Soft Computing*, 13, pp. 1055-1061, (2009).
- [4] Grzegorzewski P, Ziembińska P, Spearman's rank correlation coefficient for vague preferences, *LNAI* 7022, pp. 342-353 (2011).
- [5] Hong D.H., Kim C., A note on similarity measures between vague sets and between elements, *Information Sciences* 115, pp. 83-96 (1999).
- [6] Montes I., Janiš V., Montes S., Local IF-divergences, In: S. Greco et al. (Eds.), *IPMU 2012*, *Part II*, *CCIS* 298, Springer, pp. 491-500 (2012).
- [7] Montes S., Iglesias T., Janiš V., Montes I., A common framework for some comparison measures of IF-sets, In: *IWIFSGN 2012*, Warsaw (2012).

## An interval programming approach for an operational transportation planning problem

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**Keywords:** OR in agriculture, single-period multi-trip transportation planning problem, interval linear programming.

Crop gathering represents one of the most important stage in agricultural sector for both agricultural cooperatives and individual farmers, due to its high cost and considerable impact on crop quality and yield. Logistics and transportation activities constitute an inherent and primordial component of the agricultural supply chain. Whilst considering the dynamic behaviour and the complexity of the crop production system, an operational transportation planning problem arising usually in a typical agricultural cooperative during the crop harvest time, is presented and discussed. More speciVcally, an interval programming model with uncertain coeXcients occurred in the right-hand side of the constraints

and the objective function is developed for a single-period multi-trip planning of a heterogeneous Weet of vehicles under stochastic seed storage requests, represented as interval numbers. Best and worst optimum criteria, designed to deal with uncertainty on objective function, have been considered for both best and worst optimal solution problems, which in turn address uncertainty on right-hand side coeXcients of the constraints. SpeciVcally developed for a French Agricultural Cooperative Society situated in the region of Arcis-sur-Aube, the obtained computational results exhibit good alternative solutions and oUer a pertinent decision support by taking implicitly into account weather and farmers' delivery daily uncertainties.

### SCIENTIFIC REPORT

## Utility-Based Approach to Represent Agents' Conversational Preferences

Kaouther Bouzouita, Wided Lejouad Chaari and Moncef Tagina

Our work falls within the scope of evaluating multi-agent systems. It is inspired from the consumer economic theory since an analogy can be established with the agent model. Interactions between agents are considered in the measurement of the system utility.

With the growing interest in Multi-Agent Systems (MAS) based solutions, one can find multiple MAS conceptions and implementations dedicated to the same goal. Those systems with their complex behaviors are rarely predictable. They may provide different results according to agents' interactions sequences. Consequently, evaluation of the qual-

ity of MAS returned results became an urgent need. Our approach is interested in evaluating high level data by considering agent's preferences regarding performatives. By analogy with the economic field, agents may ask for services, so they are consumers and may receive different possible answers to their requests from other agents which are producers. We will then focus on the analysis of messages exchanged within standard interaction protocols and compute the utility value associated to every conversation. Then we conclude utility measures for each agent and for the whole MAS regarding some execution results.

### Tableau Calculus for basic fuzzy logic BL

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Basic fuzzy logic BL is a propositional logic in which the truth values of formulas are from interval [0, 1], and was introduced by Petr Hájek in [7] as a generalisation of Łukasiewicz logic, Product logic and Gödel logic. Formulas of BL are written with propositional atoms and 0 (falsum) joined by strong conjunction & and implication  $\rightarrow$ . The other connectives  $\land$ ,  $\lor$ ,  $\longleftrightarrow$  and formula  $\bar{1}(\text{verum})$  may be treated as abbreviations. The semantics of the connectives &,  $\rightarrow$  are given by a continuous t-norm and its residuum. A t-norm is a function  $\star : [0,1]^2 \rightarrow [0,1]$  that is associative, commutative and monotonic with  $x \star 1 = x$  for  $x \in [0, 1]$ . Its residuum is  $\Rightarrow$ :  $[0,1]^2 \rightarrow [0,1]$  defined by  $x \Rightarrow y = \sup\{z : x \star z \le y\}$  for  $x, y \in [0, 1]$ . Basic fuzzy logic BL has been proven in [3] and [6] to be the logic of continuous t-norms. Here, we take a further step and construct a tableau calculus for BL to prove validity of a formula or to construct a model if a formula is not valid.

A tableau calculus is a formal proof procedure that can be viewed as a refutational procedure to prove a given formula valid in a given logic, or as a search procedure for models with certain properties, in which a given formula is satisfiable, or both. In general, we start the procedure of tableau calculus with some expression, which asserts that a given formula  $\psi$  is not valid. In the case of fuzzy logic BL, we will begin with an inequality  $\Upsilon$  stating that the translated formula (to be described in full later in the paper) is less than 1. Then we apply some branch expansion rules, and either close all the branches using some closing rules, by which we fail to find a model that satisfies  $\Upsilon$ , or we obtain at least one open branch, by which we find a partial description of a model such that  $\Upsilon$  is not contradictory in it. Thus, either we prove validity of  $\psi$  by refutation, or we find a model in which  $\psi$  is not valid

Tableau methods for propositional logics have a long tradition (see [1, 5]) and have advantages over Hilbert systems such as bounded non-determinism, which means that there is a finite number of choices given by each branch expansion rule, and also the process of refutation and model searching terminates. They are automatic and thus well-suited for computer implementation. As described in the previous paragraph, not only can they show that a formula is valid, but in case it is not, they provide a model, in which the formula is not valid. There are some dual tableaux introduced for many-valued logics and fuzzy logics in [11] and [12]; however our approach is different. It follows in some respect the idea of the paper [8], where we constructed a relational hypersequents framework for Łukasiewicz modal fuzzy logic, but this approach is more general as far as the propositional rules are concerned since it deals the logic of any continuous t-norm, not only that of Łukasiewicz t-norm. There are some approaches to construct proof systems based on a relational hypersequents frameworks for BL (see [2, 13]), though our approach involves inequalities that may be implemented with some adaptation of methodology to deal with operations on real numbers as in [14], and which exist due to occurance of Product proto-t-norms in some ordinal sums. It should be noted that there are no fast and modern theorem provers and this limits real applications of fuzzy logics. There is also a gap for implementation of ordinal sums that contains Product proto-t-norms (see below). [14] We take a step towards filling this gap by introducing a tableau calculus for BL that is adaptable to logics that are complete with respect to any ordinal sums of Łukasiewicz and Product proto-t-norms.

The tableau calculus that is introduced in this paper is based on the decomposition theorem for continuous t-norms (see [4, 7, 9, 10]) by which these operations are shown to be equivalent to the ordinal sum of a family of t-norms defined on countably many intervals. We show that the calculus is sound and complete with respect to continuous t-norms, and demonstrate the refutational procedure and the search for models procedure on a selected example.

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### References

- [1] Annellis, I.: From semantic tableaux to Smullyan trees: the history of the falsifiability tree method. Modern Logic 1, 1, pp.36-69. (1990)
- [2] Bova, S, Montagna, F.: Proof Search in Hájek's Basic Logic. ACM Transactions on Computational Logic 9(3), Article 21. (June 2008)
- [3] Cignoli, R., Esteva, F., Godo, L., Torrens A.: Basic Fuzzy Logic is the logic of continuous t-norms and their residua. Soft Computing 4, pp.106-112. (2000)
- [4] Cintula, P., Hájek, P., Noguera, C.: Handbook of Mathematical Fuzzy Logic. Volumes 1 and 2. College Publications. (2011)
- [5] Fitting, M.: Introduction. In: D'Agostino, M., Gabbay, D. M., Hähnle, R., Posegga J. (eds.) Handbook of Tableau Methods. pp.1-44. Kluwer Academic Publishers. (1999)
- [6] Hájek, P.: Basic fuzzy logic and BL-algebras. Soft Computing 2, pp.124-128. (1998)

- [7] Hájek, P.: Metamathematics of Fuzzy Logic. Kluwer Academic Publishers. (1998)
- [8] Kułacka, A., Pattinson, D., Schröder L.: Syntactic Labelled Tableaux for Lukasiewicz Fuzzy ALC. In: IJCAI. (2013)
- [9] Metcalfe, G., Olivetti, N., Gabbay, D.: Proof Theory for Fuzzy Logics. Springer. (2009)
- [10] Mostert, P. S., Shields, A. L.: On the structure of semigroups on a compact manifold with boundary. Annals of Mathematics 65, pp.117-144. (1957)

- [11] Olivetti N.: Tableaux for Lukasiewicz Infinite-valued Logic. Studia Logica 73, pp.81-111. (2003)
- [12] Orłowska, E., Golińska-Pilarek, J.: Dual Tableaux: Foundations, Methodology, Case Studies. Springer. (2011)
- [13] Vetterlein, T.: Analytic Calculi for Logics of Ordinal Multiples of Standard t-Norms. Journal of Logic and Computation 18(1), pp.35-57. (2007)
- [14] Vidal, A., Bou, F., Godo, L.: An STM-Based Solver for Continuous t-norm Based Logics. In: E. Hüllermeier et al. (eds.) SUM 2012, LNAI 7520, pp.633-640. (2012)

## The Impact of T-norm Choice on Fuzzy Association Analysis

### Pavel Rusnok

Association rule mining was well established in nineties [1] as a subfield of data mining. It had been also studied much earlier in more general setting under the name GUHA [3]. Orginally the methods were developed for boolean variables. One of many generalizations proposed was to define fuzzy sets on the variable domain and mine fuzzy associations from data. The result then depends on a chosen t-norm that is used for calculating confidence and support measures of mined association rules.

Related work was published in two articles [6, 4]. In the first article only the rules with one antecedent were studied and in the second only minimum t-norm was used.

We experimented with various datasets from [2] and partitioning methods to examine relationships of mined rules by different t-norms. We measured the distances of sets of mined rules with different t-norms and also with set of rules mined by crisp association analysis.

Rules mined with the Łukasiewicz t-norm tend to be closer to crisp association rules than the rules mined by other t-norms in case of equi-width partitioning. In equi-width partitioning we are more likely to encounter the bordering effect described e.g. in [5]. The Łukasiewicz t-norm reduces the bordering effect as values near 0.5 are mapped near 0.

Increasing of the number of antecedents lead to an increase of the distances, but not in the same extent for minimum and product t-norm that tend to be still close enough.

We found out that in certain cases there are almost no common rules mined with two different t-norms (e.g. product and Łukasiewicz t-norm). Having in mind such a crucial impact of the choice of a t-norm on the results of fuzzy association analysis, the claims that fuzzy association analysis gives the same results as its classical counterpart and is useless (see [6]) simply cannot be valid and even more, they do not make sense.

For more detailed description of our results please refer to [7].

### References

- [1] Agrawal, R., Imieliński, T., Swami, A.: Mining association rules between sets of items in large databases. Proceedings of the 1993 ACM SIGMOD International Conference on Management of Data, 207–216 (1993)
- [2] Bache, K., Lichman, M.: UCI machine learning repository (2013), http://archive.ics.uci.edu/ml
- [3] Hájek, P., Havránek, T.: Mechanizing Hypothesis Formation (Mathematical Foundations for a General Theory). Springer-Verlag (1978)
- [4] Hullermeier, E., Yi, Y.: In defense of fuzzy association analysis. IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics 37(4), 1039–1043 (2007)
- [5] Sudkamp, T.: Examples, counterexamples, and measuring fuzzy associations. Fuzzy Sets and Systems 149(1), 57–71 (2005)
- [6] Verlinde, H., De Cock, M., Boute, R.: Fuzzy versus quantitative association rules: a fair data-driven comparison. Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on 36(3), 679–684 (2005)
- [7] Kupka, J., Rusnok P.: The Role of a T-norm and Partitioning in Fuzzy Association Analysis. Proceedings of the SMPS 2014, Advances in Intelligent Systems and Computing, Volume 315, 283–291 (2015)

### **GENERAL ASSEMBLY**

# Minutes of the EUSFLAT General Assembly 2014

Montpellier, July 16, 2014

The Assembly starts at 17:15, fifteen minutes later than as scheduled because of the necessity to change the room.

### Agenda:

- 1. EUSFLAT Board report
- Agreements with other associations and related conferences
- 3. EUSFLAT grants
- 4. EUSFLAT Awards
- 5. Working groups \*
- 6. Mathware & SC magazine
- 7. IJCIS
- 8. Treasury report \*
- 9. EUSFLAT 2015 conference
- 10. European Summer School on Fuzzy Logic and Applications proposal
- 11. Other matters

### \* Requires a decision

Before the official agenda of the EUSFLAT Board, Gabriella Pasi expresses a gratitude to Elie Sanchez, great scientist and pioneer of fuzzy logic, who has unfortunately passed away recently.

### 1.- EUSFLAT Board Report.

President: Gabriella PASI, Italy

Vice President: Bernard DE BAETS, Belgium Secretary: Martin ŠTĚPNIČKA, Czech Republic Treasurer: Edurne BARRENECHEA, Spain

Societes and conferences: Javier MONTERO, Spain Connections with ECSC: José María ALONSO, Spain

Connections with IJCIS: Jie LU, Australia

EUSFLAT Magazine: Humberto BUSTINCE, Spain Link to European projects: Vicenc TORRA, Spain Link to industry: Rudolf KRUSE, Germany

Recruiting: Marie-Jeanne LESOT, France Web Coordination: Jorge CASILLAS, Spain

Grants: Brunella GERLA, Italy

Working Groups: Slawomir ZADROZNY, Poland

Awards: Eulalia SZMIDT, Poland

**Gabriella Pasi**, the President of EUSFLAT announces that Janusz Kacprzyk has been elected Fellow of the European Coordinating Committee for Artificial Intelligence (ECCAI). She congratulates Janusz Kacprzyk, and she reminds that also other EUSFLAT members have received such a prestigious Award

Janusz Kacprzyk expresses his thanks to the community, and he also cites and thanks the other members of the association who received such an award, and who are attending the Assembly.

EUSFLAT membership stats were reported. The main number Ű the current number of members Ű equals to 245. The membership is distributed over 28 countries with leading Spain (106), followed by the Czech Republic (24), France (21) and Italy (18). In order to stabilize the membership numbers, Gabriella Pasi emphasizes all the membership benefits, such as reduced conference fee, access to IJCIS journal, student travel grants just to mention some of them in order to motivate people to join and mainly, in order to motivate members to help to advertise the society among their colleagues. She also mentions that an agreement with the supported conferences could be made to the aim of offering a registration fee that includes the EUSFLAT membership fee.

### 2.- Agreements with Other Associations and Related Conferences

Gabriella Pasi recalls the existing agreements with other associations that provide EUSFLAT members with the benefit of membership fees or conference fees discounts, namely HFA, NAFIPS, NSAIS, RAFSSoftCom, SIGEF, ACIA, SBA, IRSS

Exhaustive list of supported and endorsed conference is provided as well.

Call to apply for new supported/endorsed conferences!

### 3.- EUSFLAT Grants.

Brunella Gerla provides the society with a short report on student grants. From eight to twelve grants per each year are being awarded as a general society strategy with a support of the EUSFLAT conference in odd years and a support of IPMU, SMPS, LFA, MDAI and ISCAMI in even years. In 2014, the society has assigned 11 grants (5 for IPMU, 3 for SMPS, 3 for LFA).

The formal rules including the process of an application and instructions for applicants are published on the society web page http://eusflat.org. The rules for conference organizers are being specified as well, the main one that has to be emphasized, is the necessity to offer reduced conference fees to our society members.

Newly, **Brunella Gerla** introduces a proposal to ask students Ű grant holders, to submit an abstract or a scientific report to our society magazine Mathware & SC.

#### 4.- EUSFLAT Awards.

Gabriella Pasi recalls previously awarded awards, particularly, EUSFLAT Scientific Excellence Award (Didier Dubois, Petr Hájek). These awardees, together with all past and current EUSFLAT presidents will constitute a committee to determine the next EUSFLAT Scientific Excellence Award winner that will be publically announced on EUSFLAT 2015 conference.

Furthermore, **Gabriella Pasi** recalls the **EUSFLAT Best PhD Thesis Awards** that will be awarded next year. The deadline for a submission of an application is six months! before the EUSFLAT conference (June 30 Ű July 3), i.e., the deadline is January 1, 2015!

All details are published on http://eusflat.org/awards.php

Gabriella Pasi also recalls the **EUSFLAT Best student paper Award at EUSFLAT conferences** that will be also awarded next year. The deadline for a submission of an application is one month! before the EUSFLAT conference (June 30 Ű July 3), i.e., the deadline is May 30, 2015!

All details are also published on http://eusflat.org/awards.php

#### 5.- Working Groups.

Slawomir Zadrozny provides the society with a report on Working group activities. Firstly, he mentions the main features and goals of a working group, secondly he recalls the most typical activities of working groups, thirdly, Slawomir Zadrozny provides the society with new ideas (rules) in order to strengthen the activity of working groups. Slawomir Zadrozny emphasizes also the activities strengthening the visibility and mainly providing the scientific quality guarantee for both, the society as well as public audience (ontology, organization of the knowledge, Wikipedia, scholarpedia etc.). Slawomir Zadrozny also proposes the idea of asking Working Group coordinators to edit a book reporting on the scientific outcomes of the research activities undertaken by their members.

Furthermore, **Slawomir Zadrozny** provides a review of each working group activity. Finally, the announcement of the request for creating a new *Working Group on Intuitionistic Fuzzy Sets: Theory, Applications and Related Topics*(IFSTART) is provided.

### 6.- Mathware & SC Magazine.

Short report on Mathware & SCmagazine is provided by **Gabriella Pasi**.

Call for contributions, namely scientific reports, summaries of forums, panels, news and calls (PhD thesis, books, comments, events) is emphasized!

Contributions may be submitted via http://eusflat.org/msc or using the editor-in-chief email address bustince@unavarra.es.

#### 7.- IJCIS.

Gabriella Pasi provides the society with a report on the IJCIS journal and its activities. The acceptance rate has decreased last years as well as the self-citation percentage (only

4% in 2013!), which confirms that the recently introduced initiatives and actions were very positive. The hypothetical IF in 2013 calculated based on the data provided by Thomson Reuters Web of Science is 0.566. We expect to get the official IF back in 2014. However, observing the numbers of published paper by countries, the number of papers submitted by EUSFLAT members should increase.

Targets for 2015 are as follows: establish some research topics as flagships; obtained and improved IF; improvement in the online submission system; improvement of the reviewer database; improvement in the promotion for EUSFLAT (bulletin for each new issue).

### 8.- Treasury Reports: 2013 Budget, 2014 Estimated Budget, 2015 Proposed Budget.

**Gabriella Pasi** informed about the treasury reports. The main features were as follows.

2013 budget surplus: 3,790.04 EUR

December 31, 2013 account situation: 32,062.81 EUR

2014 expected budget surplus: -164.5 EUR 2015 proposed budget surplus: -870.00 EUR

December 31, 2015 proposed account situation: 31,028.31 EUR

Marcin Detyniecki asks about the non-balanced expenses of the society as the biggest portion of the budget is spent for grants. Is this coherent with the overall strategy of the society? Gabriella Pasi answers that it is, as the students mean future for the society. In this context, Irina Perfilieva asks about a competition that determines grant holders Ü is there any? Brunella Gerla and Gabriella Pasi outline that the number of applications was balanced with respect to the number of available grants. Vilém Novák suggests increasing the expenses for student grants even in order to obtain a negative surplus of year budget as there is enough money on the society bank account which can be spent in this direction which may bring better benefits than to keep them in the bank.

The 2013 treasury report, the 2014 estimated budget and finally the 2015 proposed budget were approved by all EUSFLAT members participating on the Assembly.

### 9.- IFSA/EUSFLAT 2015 Conference.

José María Alonso, presented a few slides about the organization of the IFSA/EUSFLAT 2015 conference. He presented many details as well as motivations to participate on the joint IFSA-EUSFLAT event. The main information consists in the dates of the conference (June 30 Ű July 3) and the location which will be Gijón (Asturias, Spain). Society members are advised to visit:

http://www.softcomputing.es/ifsa-eusflat2015/

Furthermore, José María Alonso recalls the main deadlines (special session proposal Ű November 1, 2015; paper submission deadline Ű January 16, 2015) and the main features motivating any interested scientist to come and participate.

### 10.- European Summer School on FuzzyLogic and Applications Proposal.

Gabriella Pasi presents the recent Board initiative to establish a regular European Summer School on Fuzzy Logic and Applications organized every year in a different place. The expected duration would be 5 days and the target would be to address PhD students, post-docs and professionals with the high-level education of both theoretical and practical aspects of fuzzy logic (in a broad sense) and of related applications. For 2015, the school could be organised in Italy in the first half of September, and in this case Gabriella Pasi herself would be the person responsible for this first year organization.

**Gabriella Pasi** formulates three crucial questions that the Board needs in order to accept or deny the decision about the initiative:

- a) How many of you would financially support their students to attend such a school?
- b) How many of you would be willing to organize such an event in the future?
- c) How many of you would agree on the organization of such an event?

There was a common positive answer to all three questions however, the number of raised hands in the first two cases was obviously significantly lower than in the case of question c). Up to some extent, this is a natural consequent of the fact that only the "heads" of departments or research groups answered to a) and b) in order to avoid misleading duplicate answers form more people from the same group, up to some extent, it is a consequence of the clear fact, that the support (question c) ) does not require to overcome obstacles as the answer to questions a) and b), as pointed out by Vilém Novák. He mentioned that he has no problem to support participation of students if it will be legal from the local point of view (local country law, university directive, project provider rules etc.) which might be a problem as most of students have no employee contracts at universities and thus, cannot be directly supported from their departments.

**Didier Dubois** supports the idea and outlines the very general and possibly misleading meaning of the expression "Fuzzy Logic"; to avoid ambiguities it proposes to leave as a general title "Fuzzy Logic", and to introduce a subtitle that may characterize a more focused topic for each summer school in order to better aim at a target group.

**Janusz Kacprzyk** suggests modifying the name ofthe summer school (avoiding the term "fuzzy logic"). Furthermore, he suggests moving the intended topic more towards applications.

**Irina Perfilieva** supports the idea of the summer school and suggests introducing an expectation that if a teacher participates on the summer school he/she will begin at least two

students with him/her. This can be done e.g. by paying a fee from the teacher and use this money to donate the students.

**Marcin Detyniecki** supports the proposed balance of theory and practice. Moreover, he supports to use some of the student grants to be spent in this direction.

**Gabriella Pasi** proposes to define an advisory board to evaluate the quality of the scientific contents of the school that will be proposed by the possible coordinators of the school in their bids for application.

During this point, some of the assembly attendants left the room.

#### 11.- Other Matters.

The newly suggested *Working Group on Intuitionistic Fuzzy Sets: Theory, Applications and Related Topics*(IFSTART) needs an approval according to Bylaws, Chapter VI, Article 29.

Before the formal approval, **Vilém Novák** opens the question of the terminological difficulties related to the working group title. This opens a rather broader discussion among the attendants.

During this point, some of the assembly attendants left the room.

Martin Štěpnička summarizes that we have two options at this moment, either to open the approval voting right now in the given setting including the proposed title of the working group, or to open the conditional approval voting with the condition to modify the name, which is on the other hand hard to accept as neither the coordinator nor any of the associative coordinators are present to confirm their willingness to consider title modification.

The rather broader discussion among some attendants continues and during this point, some of the assembly attendants leave the room.

With respect to the given situation, when most of the members already left the room and the number of the Assembly attendants is only about 10% of the overall number of EUSFLAT members and in order proceed all approvals required by Bylaws with a sufficiently strong and unquestionable mandate, Martin Štěpnička suggest to make a decision if the approval should be done on this Assembly, or rather postponed to the next one in 2015, in which a much higher number of attendants is expected. This option would moreover provide time to clarify the terminological clash.

Four attendants voted against the decision postpone, three attendants abstained from voting, vast majority agreed on postponing the decision to the next Assembly in order to accompany the decision with a strong mandate.

Society was reminded that the next EUSFLAT Assembly will take place during EUSFLAT conference 2015 in Gijón, Spain, June 30 Ű July 3.

With no other issue, the Assembly finished at 18:50.

# IPMU 2014, 15th Information Processing and Management of Uncertainty Conference



The 15th IPMU (Information Processing and Management of Uncertainty) conference, organized by Anne Laurent (Université Montpellier 2, France), Olivier Strauss (Université Montpellier 2, France), Bernadette Bouchon-Meunier (Université Pierre et Marie Curie, France) and Ronald R. Yager (Iowa College, USA), was held from 15 to 19 July 2014 in Montpellier (France). It put together around 260 researchers from the five continents.

The conference was dedicated to Professor Elie Sanchez who passed away on March 6, 2014. Professor Elie Sanchez was a pioneer in fuzzy sets and systems, not only in France where he lived and where he was one of the founders of the fuzzy community, but also in the world. Professor Elie Sanchez will be remembered for his scientific achievements, his charisma and his exceptional human qualities.



The conference participants enjoyed the brilliant plenary lectures of the invited speakers Vladimir Naumovich Vapnik (NEC Laboratories in Princeton, New Jersey (USA), Royal Holloway, University of London (UK) and Columbia University, New York (USA), "Learning with nontrivial teacher"), Marcin Detyniecki (CNRS, Sorbonne Universités, UPMC (France), Polish Academy of Sciences, Warsaw (Poland),

"New challenges in fuzzy reasoning: when practice and theory meet"), Inés Couso (University of Oviedo, Spain, "Preference relations and families of probabilities: different sides of the same coin"), Nadia Berthouze (University College London (UK), "What does your body tell me... Oh, ... and what does it tell you?"), Stuart Russell (University of California, Berkeley (USA), Université Pierre et Marie Curie (France), "Unifying Logic and Probability: A New Dawn for AI?"), Christian de Sainte Marie (IBM France, "Making decisions under uncertainty").



Professor Vladimir Naumovich VAPNIK has been selected for the Kampé de Fériet Award for his pioneering work on Statistical Learning Theory and his development of the theory of the Support Vector Machine algorithm.

Around 180 contributed papers were presented, authored by researchers from 46 countries, including the regular track and 19 special sessions: Decision Support and uncertainty management in Agri-Environment, Aggregation Functions, Big Data - the Role of Fuzzy Methods, Information Fusion Systems, Fuzzy Measures and Integrals, Soft Computing in Image Processing, Intelligent Databases and Information Systems, Intelligent Measurement and Control for Nonlinear Systems, Imprecise Probabilities: From Foundations To Applications, Management of Uncertainty in Social Networks, Fuzzy implication functions, Soft Computing and Sensory Analysis, Uncertainty and Imprecision on the Web of Data, Uncertainty Management in Machine Learning, Fuzzy logic in Boolean framework, Philosophy and History of Soft Computing, Fuzzy Logic, Formal Concept Analysis and Rough Sets, Formal Methods for Vagueness and Uncertainty in a Many-Valued Realm, From different to same, from imitation to analogy: the human and machine processing of uncertain and incomplete data for diagnosis and understanding, Fuzzy implications, Clustering, Nonclassical logics, Data analysis, Real-world applications, Aggregation, Probabilistic networks, Recommendation systems and social networks, Fuzzy systems, Database systems, Fuzzy set theory, Sensory information, Optimization, Preferences, Similarity analysis, Theory of evidence, Graduality, Argumentation.



All the works have been published by Springer in three volumes of a book of the series Communications in Computer and Information Science that are already available.

In addition, three panels were organized on topics at the core of the conference scope and the Saturday morning session was focused on real world applications.



On Tuesday, July 15, the participants had the pleasure to

enjoy a Welcome reception at the New Montpellier City Hall designed by J. Nouvel and F. Fontès, an absolute must-see for fans of fine modern architecture!

The EUSFLAT General Assembly was held on Wednesday, July 16.

On Thursday afternoon the participants joined a tour during which they had the opportunity to visit Pezenas, an old village with many historical edifices, nice narrow streets, and a rich heritage for theatre. After the excursion, the social dinner was hold in the Abbaye de Valmagne, a gothic abbey which was, from the 12th to the 13th century, one of the richest abbeys in the South of France. The participants have enjoyed a wonderful concert given by a trio.



The three Eusflat Grants, consisting in 5 grants of 300 euros each to support students for attending the IPMU 2014 Conference, were assigned from the Eusflat Society to the following young researchers: Siwar Jendoubi, Agnieszka Kulacka, Kaouther Bouzouita, Samir Hachour, Valeria Borodin.

The closing session on Friday ended the 15th IPMU conference.

### CONFERENCE REPORT

### FLINS-ISKE 2014, 11th International FLINS Conference on Decision Making and Soft Computing and 9th International Conference on Intelligent Systems and Knowledge Engineering

The 11th edition of the International FLINS Conference on Decision Making and Soft Computing (FLINS 2014) and the 9th International Conference on Intelligent Systems and Knowledge Engineering (ISKE 2014) were held in August 17-20, 2014, João Pessoa, Brazil. Those Conferences brought together researchers and students actively involved in areas of interest to the computational intelligence and its applications, to report on up-to-the-minute innovations and developments, to summarize the state-of-the-art, and to exchange

ideas and advances in all aspects of computational intelligence.

FLINS 2014 was organized by Federal University of Paraíba, Brazil and cosponsored by the Belgian Nuclear Research Centre (SCKoCEN) and Brazilian Federal Agency for Support and Evaluation of Graduate Education (CAPES). It was supported by European Society for Fuzzy Logic and Technology (EUSFLAT) and two Graduation Programs: on Decision Models and Health and on Informatics. FLINS 2014

was the eleventh in a series of conferences on computational intelligence systems. The principal mission of FLINS is bridging the gap between machine intelligence and real complex systems via joint research between universities and international research institutions, encouraging interdisciplinary research and bringing multidiscipline researchers together. FLINS2014 was held simultaneously with the 9th International Conference on Intelligent Systems and Knowledge Engineering (ISKE 2014) and the III Brazilian Congress on Fuzzy Systems (III CBSF).



**Opening Session** 

FLINS 2014 and ISKE 2014 were the first FLINS and ISKE conferences in the Americas and 122 papers from 21 countries were presented. 34.9% of the submissions were from China; 20.2% and 10.9% of the submissions came from Brazil and Spain, respectively. The congress was organized in 21 scientific sessions, 3 poster sessions, 4 invited speakers, a Doctoral Consortium chaired by Irina Perfilieva, Etienne Kerre, Yang Xu and a Round Table on Bipolarity in Social Sciences and Mathematics chaired by Javier Montero, Didier Dubois and Camilo Franco.

The submitted papers for both conferences have been published in the World Scientific proceedings under seven main titles: Part1. Decision Making and Decision Support Systems, Part 2. Statistics, Data Analysis and Data Mining, Part 3. Foundations of Computational Intelligence, Part 4. Soft Computing and Applied Research, Part 5. Intelligent Systems and Knowledge Engineering, Part 6. Uncertainty Modeling and Part 7. Intelligent Information Processing.



#### Da Ruan Award was given to Prof. Cengiz Kahraman

FLINS/ISKE 2014 had four invited speeches, which were made by Didier Dubois from France, Witold Pedrycz from Canada, Peter Sussner from Brazil and Weldon A. Lodwick from USA. The FLINS Steering Committee decided to give the Memorial Da Ruan award (founder of the FLINS Conferences, Prof. Da Ruan suddenly passed away in July 31, 2011) to Prof. Cengiz Kahraman from Istanbul Technical University, Turkey. The FLINS 2014 best paper award was given to the paper entitled "Non-Clausal Multi-Ary Alpha-Generalized Resolution Principle for a Lattice-Valued Propositional Logic" authored by Yang Xu, Jun Liu, Xingxing He and Shuwei Chen. The ISKE 2014 Best Paper award was given to the paper entitled "A Novel-Weighting Method for Online Ensemble Learning with the Presence of Concept Drift" authored by Anjin Liu, Guangquan Zhang and Jie Lu.



FLINS 2014 Gala Dinner

Three special issues of international journals were devoted for FLINS/ISKE 2014, including International Journal of Computational Intelligence Systems (SCI-E indexed, Atlantis Press), Journal of Multiple Valued Logic and Soft Computing (SCI-E indexed, Oldcity Publishing), Knowledge-Based Systems (SCI indexed, Elsevier).

The next edition of FLINS conference will be organized by Prof. Xianyi Zeng from University of Lili and will be held in Roubaix, France in August, 2016.



# CBSF 2014, II Brazilian Congress on Fuzzy Systems





Since 2006 the Brazilian research community around the theme "Fuzzy" have organized meetings in order to discuss it. The pionner event was the Symposium on Applications of Fuzzy Logic (Simpósio de Aplicações de Lógica Fuzzy - SALF - in Portuguese), for which it were organized two events. Due to the increasing number of attendents (students and researchers) from several states of the country.

The CBSF expanded the scope of SALF, which focused mainly on applications. The word "System" assumed a broad sense, in order to capture researches on Computational Systems and on Logical Systems. The congress became the first event in Latin America about this theme and thus it was supported by several scientific societies, since the beginning.



In 2009, during the National Congress on Applied Mathematics, the Brazilian Applied Mathematical Society (SBMAC) promoted the First Mini-symposium on the "Foundations and Applications of Fuzzy Logic" as a satellite event of the first one. A Thematic Committee on Fuzzy Systems was created in the SBMAC and the First Brazilian Congress on Fuzzy Systems (I Congresso Brasileiro em Sistemas Fuzzy - I CBSF; in Portuguese) was created in replacement to the III SALF which

was been planned.



The III CBSF was held in João Pessoa - PB (Brazil) in August, 17 - 20th, 2014, organized by the Federal University of Paraíba. It was organized by Ronei Marcos de Moraes (General Chair) and Liliane S. Machado(General Co-Chair), in colaboration with Regivan Hugo Nunes Santiago (PC Chair) and Wladimir Seixas (PC Co-Chair). The III CBSF was supported by the following scientific societies:

- Brazilian Applied Mathematical Society (SBMAC)
- Brazilian Society of Automaticac Control (SBA),
- Brazilian Society of Computational Intelligence (SBIC),

- International Fuzzy Systems Association (IFSA),
- European Society for Fuzzy Logic and Technology (EUSFLAT),
- North American Fuzzy Information Processing Society (NAFIPS).



The III CBSF was held simultaneously with the 11th International FLINS Conference on Decision Making and Soft Computing (FLINS2014) and the 9th International Conference on Intelligent Systems and Knowledge Engineering (ISKE 2014). The congress was organized in 6 plenary sessions, 2 minicourses, 16 technical sessions and a Round Table on Bipolarity in Social Sciences and Mathematics chaired by Javier Montero, Didier Dubois and Camilo Franco. The topics of the meeting were divided into two categories Theoretical and Applied aspects of Fuzzy Systems. Two minicourses were provided; namely: "Introduction to the arithmetic of fuzzy relational algebra" (Petrucio Viana) and "Introduction to Fuzzy Differential Equations: Differential inclusions versus Zadeh's Extension" (Marina Tuyako Mizukoshi). The six plenary talks were:



- Fuzzy Systems: Theoretical and Practical Aspects (Laecio Carvalho de Barros)
- Generalizations of Fuzzy Sets Theory (Benjamín Rene Callejas Bedregal)
- The Contribution of Fuzzy Sets in Decision Sciences (Didier Dubois)

- Granular Fuzzy Systems: A New Direction in Soft Computing and Human-Centric Decision-Making (Witold Pedrycz)
- Some Approaches towards Lattice Computing in Mathematical Morphology and Computational Intelligence (Peter Sussner)
- Solving Linear Systems with Uncertain Coefficients: An Unified Approach (Weldon A. Lodwick)



Although most of the contributions came from the national groups, we could find some contributions from South America and some other countries. The proceedings were published by the Brazilian Society for Applied and Computational Mathematics (SBMAC) and can be downloaded from III CBSF Webpages at https://sites.google.com/site/brazilianfuzzy/. The auhors of the best papers of the conference will also be invited to submitt an extended version to the next issue of Mathware and Soft Computing Magazine.



Some other pictures can be found in http://www.facebook.com/congressobrasileirodesistemasfuzzy. The next edition of Brazilian Congress on Fuzzy Systems will be held in Campinas -SP (Brazil).

### **SEMATICA 2014**

SEMÁTICA is an annual seminar for Computer Sciences Ph.D. students in Spain. SEMÁTICA seminars are organized by the ÁTICA research network, devoted to the Advance and Transference of Applied Computational Intelligence (see http://www.cs.upc.edu/atica/).

The 4th SEMÁTICA seminar (http://www.cs.upc.edu/atica/sematica2014/) was held in the Universitat Politècnica de Catalunya û BarcelonaTech (UPC, November 20-21), following the successful editions in Santiago de Compostela University (2011), Granada University (2012) and Complutense University of Madrid (2013).

During these seminars, Ph.D. students have the opportunity of presenting their work (by means of either an oral presentation or a poster) in a friendly audience with their Ph.D. colleagues and helpful senior ÁTICA researchers that promotes collaboration between Ph.D. students and other research groups. A total of 24 Ph.D. students participated in this SEMÁTICA 2014 edition.

During SEMÁTICA seminars, Ph.D. students also receive interesting formative talks that should help them in their professional strategy and the necessary transfer of their research to Society. In SEMÁTICA 2014 there were two invited speakers: Jordi Vitrià from the University of Barcelona and also belonging to the Center for Computer Vision, who gave a talk on Data Science, and Jazmín Aguado-Sierra from the Barcelona Supercomputing Center, who gave a talk on Computational Cardiology.

Beside Ph.D. student presentations and those formative talks, a third element of SEMÁTICA is the celebration of the

Three Minutes Thesis competition on Computational Intelligence (http://atica.cs.upc.edu/t3m/). The Three Minute Thesis (3MT) is a research communication competition launched by The University of Queensland (Australia) which challenges research higher degree students to present a compelling oration on their thesis and its significance in just three minutes in language appropriate to a nonspecialist audience (http://threeminutethesis.org/index.html). More than 200 Universities around the world have joined this initiative since 2008.

After an open call in Spain, 9 finalists were selected from all submitted videos, and they had to make their three minutes final presentation so a jury could decide about the awards, taking mainly into account the communicative skills of each Ph.D. student. The final contest could be followed on line by streaming. From the 9 selected finalists, finally 7 finalists participated to the contest. In this edition, the ÁTICA-T3M Award went to Luis Oliva, from the Universitat Politècnica de Catalunya, and the Knowledge Engineering and Machine Learning Group Award (KEMLG-UPC) went to Borja Vázquez, from the Universidade de Santiago de Compostela. An honorific mention (mention ÁTICA-T3M) was also acknowledged to Martha Ivón Cárdenas, from the Universitat Politècnica de Catalunya.

Miquel Sánchez-Marrè, local chair of SEMÁTICA 2014 and of the 3MT Competition on Computational Intelligence 2014, from Universitat Politècnica de Catalunya û BarcelonaTech



### FuzzyMAD 2014 Seminar

As already pointed out in some previous issues of this EUSFLAT Magazine, FuzzyMAD seminar was created in 2008, when several fuzzy researchers in Madrid realized that only on the occasion of international conferences outside Madrid they had the opportunity to meet. FuzzyMAD, now in its 7th edition, was then launched as an annual opportunity to interchange experiences and explore scientific collaborations from Ph.D. level to join funded projects that could take advantage of the great scientific potential of Madrid, with more than 14 Universities, a big number of research institutions, and a dynamic private sector. Being Madrid a busy city, the format of FuzzyMAD has evolved into an intense 5-hours meeting between 11:00 and 16:00 hours. We start as a Ph.D. Seminar, where young researchers present in a short talk the current status of their Ph.D. thesis, and then it follows an attractive less-technical talk that always pursues some transversal knowledge. We finish FuzzyMAD with a poster session in which each group presents their last achievements meanwhile we have a buffet lunch. Participants also share with colleagues those papers they consider have more potential for future collaboration, and other nonfuzzy researchers are also invited to expand possibilities for collaboration. This FuzzyMAD 2014 held on December 12, with 49 attendants, and a wonderful plenary talk of Prof. Aníbal Figuieras from Carlos III University of Madrid, about strategies to potentiate the creative thinking in research.

Javier Montero and Daniel Gómez, local chairs of Fuzzy-MAD 2014.

Complutense University, Madrid, Spain

(FuzzyMAD 2014 has been partially supported by the UCM research group GR3/14-910149 and the CASI-CAM-CM network S2013/ICE-2845 from the Government of Madrid).



### Ph.D. Thesis defended by Noelia Hernández

Department of Electronics, University of Alcala, Madrid, Spain



Noelia Hernández defended her PhD Thesis, entitled "Smart Hierarchical WiFi Localization System for Indoors", on July 22, 2014. Her advisors were Dr. Manuel Ocaña (University of Alcala) and Dr. José M. Alonso (European Centre for Soft Computing).

Recent years have seen a rapid growth of smartphone and tablet applications. Many of these applications make use of the localization capabilities of these devices in what are called Location Based Services (LBS). To be able to provide this kind of services, a reliable and real time identification of the user location is required. Traditionally, global localization has been carried out through the Global Positioning System (GPS), which provides accurate localization when working outdoors. Unfortunately, the use of GPS is affected by Non-Line-Of-Sight, making GPS localization in indoor environments not suitable.

Different technologies are being used to provide indoor localization, among them, WiFi is a common choice due to its important advantages: there are WiFi access points (APs) in most buildings and measuring WiFi signal is free of charge even for private WiFi networks. Unfortunately, it also has some disadvantages: when working indoors the

signal strength is strongly dependent on the building structure and some other non-desired effects appear, such as the multipath effect, signal absorption and the small scale variations. Moreover, since WiFi networks are deployed with the goal of maximizing connectivity and disregarding localization tasks, there are usually many APs distributed over the environment increasing the so-called co-channel interferences, which cause high variations in the received signal strength (RSS) from the APs.

The goal of this thesis is the localization of mobile devices in indoor environments using as the only available information the signal received from the already existing APs in the environment. Since WiFi is pre-installed in most of the buildings, there is no need to either modify the environment or add new devices to it. Then, the final research objective is to achieve robust WiFi real-time localization for mobile devices, available to be deployed in any environment and to be used by any device.

To achieve this objective, a hierarchical fuzzy-based approach is proposed to perform localization in topologically described environments. All RSS variations (small-scale variations, large-scale variations, temporal variations, and so on) are effectively handled by means of fuzzy sets and systems. As result, the new approach is able to deal with multi-floor large environments. To do so, the system first creates a hierarchical partition of the environment using similarity clues in a K-Means-based approach. Then, the localization system is trained using different supervised fuzzy learning algorithms to classify the new WiFi samples through the hierarchical tree of the environment partition. Finally, the system is enhanced with a Bayesian filter in charge of tracking the position of a device in motion.

The system has been tested in several real-world multifloor large environments: University of Alcalá (Spain), European Centre for Soft Computing (Spain), University of Edinburg (UK), and Yonsei University (Korea). It has yielded an overall mean error distance under 3 metres what is acceptable for most LBS applications.

For additional details, the interested reader is kindly referred to: http://www.robesafe.uah.es/personal/noelia.hernandez/research.html

# Ph.D. Thesis defended by Davide Martinetti

Department of Statistics and O.R., University of Oviedo, Spain



Davide Martinetti defended his PhD Thesis entitled "Fuzzy and probabilistic approaches to modelling individual choice and preference. Rationality conditions and their relationships" on July, 24. His thesis was supervised by Susana Montes and Susana Díaz, from the University of Oviedo and by Bernard De Baets, from the University of Ghent.

The main subject of the presented work is a comprehensive study of choice theory in the fuzzy and the probabilistic settings.

The author explored the possibility of extending the clas-

sical results of choice theory to the case in which the available alternatives, the choice function and the preference relation are considered to be fuzzy. The first important result establishes under which conditions a fuzzy preference relation can rationalize a fuzzy choice unction by means of the fuzzy set of the greatest or the maximal elements of a set with respect to a fuzzy preference relation. Properties such as acyclicity and completeness play a key-role in this context.

The second part of the work is dedicated to the extension to the fuzzy sets framework of the Arrow-Sen-Suzumura theorem, a milestone in classical choice theory. The original result established the equivalence between different rationality conditions of crisp choice functions, such as the Weak and Strong Axioms of Revealed Preference, the Weak and Strong Congruence Axioms or the expansion-contraction conditions proposed by Sen. The purpose of the study was to extend that result in the fuzzy choice context and a comprehensive set of conditions has been considered.

The third and last part of the work is aimed to study the relationships between the fuzzy and probabilistic formalization of choice theory. First of all, a construction to equate fuzzy and probabilistic choice functions is presented, making use of implication operators derived from a t-norm. Under this construction, a set of new conditions for the probabilistic choice function is proposed (Weak and Strong Scalability Conditions and Weak and Strong Stochastic Congruence Axioms) and it is proved to ensure the normality of the associated fuzzy choice function, plus regularity of the fuzzy revealed preference relation. Also the relationships between fuzzy and probabilistic preference relations is considered: a construction that make use of implication operators is used in order to equate the two definitions, that even if formally similar, are strongly different in their interpretation, due the different semantics that they exhibit. Thanks to this new construction, different types of transitivity for fuzzy and probabilistic preference relations are compared, such as transitivity w.r.t. a t-norm and cycle-transitivity.

# Ph.D. Thesis defended by José Manuel Soto-Hidalgo

Department of Computation and Artificial Intelligence Sciences, University of Granada, Spain



José Manuel Soto defended his PhD Thesis entitled "Desarrollo de Modelos Difusos para Representar la Semántica del Color" (Development of Fuzzy Models to Represent the Semantics of Color) on December, 12. His thesis was supervised by Jesús Chamorro and Daniel Sťánchez, from the University of Granada.

Color is one of the fundamental characteristics in image processing. Therefore, color modeling is an important while complex problem because, among other reasons, the color is generally imprecise (it is not possible to stablish a clear boundary that delimits color categories in general), subjective (not everyone distinguish or name in the same way the

colors) and context (the same color can have different meanings in different areas). This Thesis proposes, in general, formal models to represent the semantics of color in images (addressing the problems of imprecision, subjectivity and context dependence) and operations based on these representations (relationships between color histograms, etc.).

To do this, in the first part of the Thesis, formal definitions of the concepts of "fuzzy color" and "fuzzy color space" and their properties are introduced, which address the vagueness and subjectivity in the modelling in terms of colors. In addition, a methodology for constructing fuzzy color spaces according to the paradigm of conceptual spaces is proposed. This methodology is illustrated by designing different fuzzy color spaces from existing color naming systems and from data provided by users.

Also, measures of correspondence between color information expressed through crisp and fuzzy colors, making use of Fuzzy Logic and the Theory of Possibility, considering imprecision, uncertainty or both are proposed. Specifically similarity relations, compatibility, possibility and necessity, among others, which are necessary in order to analyze different semantic of the use of fuzzy colors.

In addition, in this Thesis, different notions of histograms on fuzzy color spaces are defined, allowing to represent the number or percentage of pixels in an image taking into account the imprecision in both the count and the color itself. For this, first, an analysis of the most used approach to address the cardinal in fuzzy sets is performed and, secondly, a new definition of linguistic histogram based in compatibility between cardinal measures and linguistic labels using fuzzy quantification techniques is proposed in order to provide understandable information to the user.

Finally, a fuzzy color descriptor based on the concept of dominance and some fuzzy comparison measures are proposed. The descriptor can face the problem of imprecision in the description of color with linguistic terms, as well as dominance. Additionally, the fuzzy descriptor and measures for comparison are shown through several examples of image retrieval based on color dominance.

# Best final project communication award won by Mikel Elkano Ilintxeta

Departamento de Automática y Computación, Universidad Publica de Navarra, Spain



Mikel Elkano Ilintxeta was awarded by the Public University of Navarra as the Best Final Project Communication in the Three Minute Thesis competition held in this university. During the event, all participants had to give an oral presentation of their works in a period of 3 minutes, being allowed to make use of a single static slide.

In this project, entitled "IVOVO: sistema de multi-clasificación basado en datos intervalo-valorados" (IVOVO: multi-classification system based on inter-valued data), authors propose to improve the performance of Fuzzy Rule Based Classifier Systems applying decomposition strategies and modelling the conjunction in fuzzy rules using n-dimensional overlap functions, also defined in this work. Additionally, One-vs-One strategy was adapted to work with Interval-Valued Fuzzy Rule Based Classifier Systems. To do so, authors present a new normalization method that allows one to normalize the intervals stored in the score matrix.

**NEWS** 

## Two new published books by Angel Garrido

Angel Garrido LOGICAS DE NUESTRO TIEMPO Editorial Dykinson Madrid, 2014



A field of current scientific research, most of the truly serious and powerful, is the Artificial Intelligence (also called

Computational). Within it have one of your most valuable weapons, Fuzzy Logic.

It is generally believed that this came from a "bright idea" of an Azerbaijani mathematician and engineer, Lotfi A. Zadeh, a professor at the University of California. But this is not so: the Many-Valued Logic rooted to Aristotle himself, with his analysis of the problem of 'future contingentst', then follow as in medieval times, with Duns Scotus and William of Ockham, crossing the famous "De Auxiliis" controversy, which Domingo Banez vs Luis de Molina, for these ideas have included the issue of human free will and Divine Foreknowledge.

After a long dark times, through a line of central European influence, passing through Leibniz, Bolzano, Brentano and Twardowski, we come to the formation of the Lvov-Warsaw School, which led to great logicians and mathematicians: Stefan Banach, creator of modern functional analysis, Jan Lukasiewicz, who is the father of many-valued logics, and his assistant, Alfred Tarski, with its Semantic Theory of Truth. Rarely human mind has reached such heights.

The ideas of Lukasiewicz, always concerned about the problem of Determinism, were quite well known by Stephen

C. Kleene, and it was through him as they reached Lotfi A. Zadeh, his friend, who saw their potential applications and enthusiastically devoted to make them known.

Also research has progressed a lot between us in recent times, due among other notable efforts to center around Enric Trillas, with remarkable groups of Navarre (UPNA), Granada, or Barcelona (CSIC).

In this work we analyze aspects of this type of reasoning, and their historical evolution. In the next book, about to appear in the same Editorial Dykinson, the entitled *Applied Logic, Vagueness and Uncertainty*, we will deal with some aspects of more strictly mathematical nature.

Angel Garrido LÓGICA APLICADA: VAGUEDAD E INCERTIDUMBRE Editorial Dykinson Madrid, 2014



Nowadays logics (and, in particular, multivalued ones) are in the intersection between at least three different knowledge areas: Philosophy, Mathematics and Computer Science.

The fact of whether the subject belongs more or less to one of these areas depends on the adopted focus and the considered questions. In this new volume we clarify, - and by the way, systemize-, Fuzzy Logics along with some others, as well as their application to Uncertain Reasoning. This has been very useful in several fields, such as Medicine, Law or Engineering, but it also becoming more and more useful in others such as those related to Humanities. Because both classical Mathematics and, specially, non-classical ones, may be used in processing problem when we move in vague or uncertain

environments. Among what they have generated we have Genetic Algorithms, Computing with Words, Fuzzy Control of Systems, Artificial Neural Networks, Fractals, Chaos Theory, etc.

Angel Garrido Bullón is a Ph.D. staff professor at the Sciences Faculty of the UNED in Madrid. He made his Ms.C. and Ph.D. at the Complutense, Polytechnics and UNED Universities in Madrid. His Ph.D. was about Mathematical Logic, with the dissertation "Philosophy and Mathematics of Vagueness and Uncertainty". He has been a teacher and researcher in the UNED, as well as in the University of Manchester and the Polytechnics University of Madrid. He has published up to now fourteen books and more than two hundred papers in prestigious international journals. Among other awards, he has received the Gold Medal of the University of Bacau and the First Birkhäuser Award to the best communication in the International Conference of Mathematics (ICM'06). He is a teacher of Logics, Mathematical Analysis and Artificial Intelligence in the different university levels.

#### The author

Angel Garrido is Full Professor of Mathematical Analysis, at Fundamental Mathematics Department, Faculty of Fundamental Sciences UNED.

He completed the degree in Pure Mathematics at the Complutense University and pursued a Ph.D. in Mathematics and Computing from UNED and Polytechnic University of Madrid. Doctor of Philosophy, with a thesis on Mathematical Logic entitled "Philosophy and Mathematics Vagueness and Uncertainty", qualified with Summa cum Laude.

He has also taught at the University of Manchester and the Polytechnic of Madrid. He is currently the Editor in Chief of the magazine *AXIOMS. MATHEMATICAL LOGIC AND MATHEMATICAL PHYSICS*, published in the city of Basel (Switzerland). He is also member of the Editorial Board of several international scientific journals (as *SYMMETRY, CENTRAL EUROPEAN COMPUTER SCIENCE, ACTA UNIVERSITATIS APULENSIS, INTERNATIONAL JOURNAL OF MATHEMATICAL ANALYSIS*, among others).

He has served on several research projects of international character, on Fuzzy Logic, Computational Intelligence, Polemics and Controversies, etc. Also he has published twelve books to date and over two hundred articles.

For his research he has received, among other awards, the Gold Medal of the University of Bacau, and Prime Birkhäuser Award for Best Presentation at the International Congress of Mathematicians in Madrid (ICM'06).

### CALLS

# IFSA-EUSFLAT 2015, World Congress of the International Fuzzy Systems Association (IFSA) and Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT)

Gijón (Spain) 30 June - 3 July 2015



The 16th World Congress of the International Fuzzy Systems Association (IFSA) and the 9th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT) will be jointly held by summer 2015 in Gijón, Asturias (Spain). The aim of this joint conference is to bring together researchers (both theoreticians and practitioners) all along the world working on fuzzy logic, fuzzy systems, soft computing and related areas. Thus, scientists, engineers, students, and professionals will discuss, exchange ideas, foster interaction between industry and academy through building multidisciplinary linkages, and disseminate the most recent advancements in the field.

The meeting will also serve to celebrate the first fifty years of fuzzy sets since the publication of the seminal paper by Prof. Lotfi A. Zadeh.

#### **Sumissions**

IFSA-EUSFLAT 2015 solicits original research contributions of theoretical and methodological nature as well as application-oriented work. Contributions will be selected based upon their quality as evaluated by at least two referees. Prospective authors are invited to prepare full papers of 6-8 pages, including results, figures and references in PDF format. Authors are responsible for ensuring that their submissions are in valid format and that they will be readable and printable.

We welcome the organization of special sessions on theoretical or application-based specific topics. Proceedings will be published by Atlantis Press. They will be available online in Open Access mode before the start of the conference. Moreover, conference papers will be indexed in DBLP, EI-Compendex and ISIProceedings.

### **Topics**

The topics of interest cover all aspects of fuzzy systems:

- Fuzzy sets and fuzzy logic
- Fuzzy inference systems
- Fuzzy decision analysis, decision making, optimization and design
- Fuzzy system architectures and hardware
- Fuzzy methods in data analysis, statistics and imprecise probability
- Fuzzy databases and information retrieval
- Fuzzy pattern recognition and image processing
- Fuzzy control
- Fuzzy preference modeling
- · Fuzzy database mining
- Interpretable fuzzy systems
- Mathematical foundations of fuzzy sets and fuzzy systems
- · Possibility theory





Approximate reasoning and soft computing:

- Evolutionary and hybrid systems
- Fuzzy neural networks
- Neuro-fuzzy systems
- Genetic-fuzzy systems
- Intelligent agents and ambient intelligence
- Learning, adaptive, and evolvable fuzzy systems
- Data mining and knowledge discovery
- Knowledge extraction, representation, and modeling
- Linguistic summarization
- Computing with perceptions and words

### Real-world applications:

Robotics and control systems

- Bioinformatics
- Industrial applications
- Management science
- Operations research and manufacturing
- · Financial forecasting
- Agriculture
- Social Sciences
- Other application domains

### **Committees**

Honorary Chairs

Lotfi A. Zadeh - Michio Sugeno - Enric Trillas

General Chair

Luis Magdalena

**Program Co-Chairs** 

Oscar Cordón - Marek Reformat - Shun-Feng Su

**Special Sessions Co-Chairs** 

Humberto Bustince - Vladik Kreinovich - Hisao Ishibuchi

Local Arrangements Chair

Jose M. Alonso

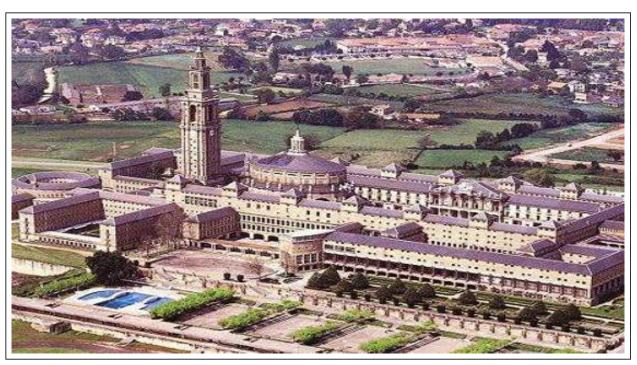
### **Important dates**

Paper submission: January 16th, 2015 Acceptance/rejection notification: March 16th, 2015 Cameraready papers: April 16th, 2015

### **Registration fees**

IFSA and EUSFLAT members will have reduced fees. In addition, selected students will be granted by the EUSFLAT Students Grant Program

For additional information please visit http://www.softcomputing.es/ifsa-eusflat2015/or contact the organizing committee at ifsa-eusflat2015@softcomputing.es



### CALLS

# ISIPTA 2015, International Symposium on Imprecise Probability: Theories and Applications

Pescara (Italy) 20-24 July 2015

Website with up-to-date information on all aspects of the conference: http://www.sipta.org/isipta15.

Scientific program contact: isipta15@easychair.org Local organization contact: isipta15@unich.it

### **Important Dates**

- 30 Jan Paper abstracts & preliminary papers due
- 13 Feb Review-ready papers due
- 31 Mar Conference hotel pre-reservation deadline
- 08 Apr Paper notification
- 17 Apr Poster-only abstracts due
- 06 May Poster-only notification
- 29 May Early-bird registration deadline (Å350 full; Ă200 student)

### **Scope & Symposium Format**

ISIPTA is the primary international forum to present and discuss new results related to imprecise probability.

We welcome both theoretical and applied original contributions. In this edition, we especially welcome papers connecting imprecise probabilities with related research in fields such as economics, philosophy, sociology, and engineering. There will be no parallel sessions. Each paper that is accepted is presented both

- (i) in a plenary session, meant for a short introduction and a sketch of the context and relevance; and
- (ii) in a poster session, where ample time is given for indepth explanation and discussion.

The poster-only track provides a second, free format presentation option. It is also open for preliminary results, challenges, etc.

### New: non-proceedings papers

To adapt to different publication cultures in different fields, we introduce a new option: the non-proceeding papers. Authors may request that their contribution is not published in the proceedings in case this precludes later publication of an expanded journal version. The PC Board will decide on a case-by-case basis. If accepted, a one-page abstract will be published in the proceedings, and the paper must be freely available on-line (e.g., as a working paper).

### **Already Confirmed Invited Speakers**

Massimo Marinacci, AXA-Bocconi Chair in Risk Department of Decision Sciences, Bocconi University, Milan, Italy

Itzhak Gilboa, Professor of Economics and Decision Sciences Eitan Berglas School of Economics, Tel-Aviv University, Israel & HEC, Paris, France

### Special Issue

After the conference, a special issue with a selection of the accepted papers will be published in an expanded version in the International Journal of Approximate Reasoning. **Steering Committee** 

- T. Augustin (LMU München, Germany) + PC Board
- G. de Cooman (Ghent University, Belgium)
- S. Doria (University G. d'Annunzio, Italy) + Local Organization
- E. Miranda (University of Oviedo, Spain) + PC Board
- E. Quaeghebeur (CWI, Amsterdam, Netherlands) + PC Board
- T. Seidenfeld (Carnegie Mellon University, USA)

We look forward to your contribution and to seeing you in Pescara!



### CALLS

# FUZZ-IEEE 2015, IEEE International Conference on Fuzzy Systems

Istanbul (Turkey) 2-5 August 2015



The 2015 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2015), one of the leading international conferences in the field of fuzzy sets and systems, will be held in Istanbul, Turkey. Istanbul is the largest city of Turkey and the third largest city in the world. The city is located on the Bosphorus Strait and encompasses the natural harbor known as the Golden Horn. It extends both on the European (Thrace) and on the Asian (Anatolia) side of the Bosphorus, and is thereby the only metropolis in the world which is situated on two continents.

### Scope

The conference will provide a platform for researchers and practitioners to interact with one another and discuss the state-of-the-art developments in the field. The topics of the conference will cover all aspects of research and applications in fuzzy systems and soft computing, including but not limited to:

- Fuzzy data analysis, fuzzy clustering, classification and pattern recognition
- Type-2 fuzzy sets, computing with words and granular computing
- Fuzzy systems with big data and cloud computing, fuzzy analytics and visualization
- Fuzzy control, robotics, sensors, fuzzy hardware and architectures
- Fuzzy systems design and optimization
- Fuzzy decision analysis, multi-criteria decision making and decision support
- Fuzzy logic and its applications in Industrial Engineering
- Fuzzy modelling, identification and fault detection
- Fuzzy information processing, information extraction and fusion

- Fuzzy web engineering, information retrieval, text mining and social network analysis
- Knowledge discovery, learning, reasoning and knowledge representation
- Fuzzy image, speech and signal processing, vision and multimedia data
- Fuzzy databases and information retrieval
- Rough sets, imprecise probabilities, possibilities approaches
- Industrial, financial, and medical applications
- Fuzzy logic application in civil engineering, geographical information systems
- Fuzzy sets and soft computing in social sciences
- Linguistic summarization, natural language processing
- Soft computing in security systems
- Software and hardware applications
- Adaptive, hierarchical, evolutionary, and neural and nature-inspired systems
- Mathematical foundations of fuzzy sets, fuzzy measures and fuzzy integrals.

In addition to regular oral and poster presentations, the conference will include a full program of tutorials, workshops, panels, special sessions and keynote presentations. Full details of the submission process will be made available on the conference website. Conference proceedings will be included in IEEE Xplore Digital Library. Authors whose papers win the best paper awards of FUZZ-IEEE 2015 will be invited to submit the enhanced/extended versions of conference papers to IEEE TFS (IEEE Transactions on Fuzzy Systems). Extended versions of selected papers will also be considered for possible publication in the special issues of several high impact journals.

### Important dates

Paper submission
February 8, 2015
Notification of acceptance for papers
March 23, 2015
Camera-ready paper submission
April 21, 2015
Early registration deadline
April 23, 2015
Conference

August 2-5, 2015

### Chairs

Honorary Co-Chairs Lofti A. Zadeh Hans R. van Nauta Lemke Frederick E. Petry General Chair Adnan Yazici General Co-Chair Nikhil R. Pal Program Chair Uzay Kaymak Program Co-Chairs Trevor Martin Hisao Ishibuchi C. T. Lin

