EUSFLAT
EUROPEAN SOCIETY FOR FUZZY LOGIC AND TECHNOLOGY

Mathware & Soft Computing

The magazine of the European Society for Fuzzy Logic and Technology

Dialogue between Etienne E. Kerre and Javier Montero

Soft Computing and Computer Vision in Forensic Identification

Abstracts of students' works granted by EUSFLAT

Selected papers from the III Brazilian Congress on Fuzzy Systems

PhD Thesis defended. News and Calls

Vol. 22, n. 1
June 2015

Mathware & Soft Computing

The magazine of the European Society for Fuzzy Logic and Technology

Volume 22, number 1
JUNE 2015

Dep. Legal: B-35.642-94
ISSN 1134-5632

`http://www.eusflat.org/msc`

Hot Topics

Scientific Reports

Mathware & Soft Computing

The magazine of the European Society for Fuzzy Logic and Technology

Volume 22, number 1
JUNE 2015

Dep. Legal: B-35.642-94
ISSN 1134-5632

http://www.eusflat.org/msc

Scientific Reports

World Echoes

Message from the Editor-in-Chief (June 2015)

HUMBERTO BUSTINCE

The new issue of our Mathware&Soft Computing magazine has arrived to your computers. And once again, we have our best to bring it full of contents which may attract the interest not only of our community but also of those working in fields which are more or less close to us.

To start, and following the tradition of all our previous issues, we publish a dialogue between two of the main relevant members of our community. In this case, Javier Montero and Etienne Kerre has accepted to share with all of us their opinions, experiences and points of view. Of course it is not necessary to introduce them, but Daniel Gómez has accepted to write a very personal view on a fuzzy day in the life of Javier Montero.

One of the main purposes of our magazine is to promote and disseminate scientific activities. In this sense, this issue comes with some relevant works that approaches us to different aspects of nowadays research. First of all, by B.R. Campomanes-Álvarez, C. Campomanes-Álvarez, E. Bermejo, A. Valssecchi, O. Ibáñez, S. Damas and O. Cordón have prepared a report on the use of Soft Computing and Vision techniques for Forensic identification. This work provides a very interesting view of an application of fuzzy techniques in a specific problem. I would like to profit from this collaboration to invite every researcher in our community to communicate its work to all the readers of our magazine. In this way I am sure many interesting collaborations between researchers will emerge.

Besides, we also publish in this issue a selection of the best papers which were presented at the Third Brazilian Congress on Fuzzy Systems, which was held in Joao Pessoa in August 2014. In particular, we include 4 papers which give us just a view of the huge development that Fuzzy Sets theory is enjoying nowadays at the other side of the Atlantic, thanks to the work of many devoted researchers and students. I have no doubt that, in the future, the Brazilian community will become very relevant and many links will be established with them. For this reason, and from our magazine, we want to help their work to be widely known.

But of course we do not forget our own EUSFLAT community, to which our work is devoted. For this reason, every announcement, call for paper or news that you have sent has been also published in this issue. Moreover, and following with a section already started in previous issues, we have also included an abstract of the contributions of students which have awarded an EUSFLAT supported grant. The very high quality of the work of these young researchers is a guarantee of future and, reading these abstracts, all of us can congratulate.

In a few days many of us will meet in Gijón, in the joint IFSA-EUSFLAT conference. I hope that this conference will open many research lines and collaborations and the pages of our Mathware&Soft Computing magazine will be open to receive all of them. Recall that this magazine is your magazine and that only with your help and collaboration, new issues will be possible. So our pages are open to any contribution you consider.

And now, enjoy our Mathware&Soft Computing magazine!

Humberto Bustince
Editor-in-chief

Message from the President (June 2015)

GABRIELLA PASI

Dear EUSFLAT members,

the date of the Joint IFSA/EUSFLAT Conference is approaching; in fact the 16^{th} World Congress of the International Fuzzy Systems Association (IFSA), and the 9^{th} Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT) will take place jointly in Gijón, Asturias, Spain, from June 30th to July 3rd, 2015. The program of the conference is extremely rich and interesting; many thanks to the organisers for the great job they did to make this event a successful event! The participation of the EUSFLAT members to this conference is very important, as it will offer an opportunity to meet colleagues from both the European and the international Fuzzy Associations, by thus offering a chance to open new collaborations and to launch new joint projects. For this reason I invite all of you warmly to attend, and to participate to the EUSFLAT General Assembly, that will take place during the IFSA/EUSFLAT Conference, on July the 1st from 6pm to 8pm (Sala Anfiteatro). During this Assembly the elections of the new Board will take place: so your participation is very important.

An important event that I am happy to announce is the first European Summer School on Fuzzy Logic and Applications, which will take place at the Lake Como School of Advanced Studies, Villa del Grumello, Como, Italy, from the 7th to the 11th of September 2015 (`http://sfla.lakecomoschool.org/`). This school is one of the outcomes of the current EUSFLAT Board, which is nearing its end. The school is aimed at PhD students, Post-Doc and also to professional to whom an introduction will be offered both to theoretical aspects and to established and emerging applications. This School has been conceived as a yearly event that will be organized each year by different research teams and Universities; the bids to apply for the organization of the school will be opened, and each EUSFLAT member can apply. We need your help to make this school a success! Please ask your students to register: they are also expected to prepare a poster synthesizing their research that will be discussed with the lecturers during the School. Please notice that EUSFLAT offers 8 grants covering the registration fee to PhD students that are EUSFLAT student members. Students who wish to apply for a grant have to send their application to the address grants@eusflat.org, by justifying their request and by including both a curriculum vitae and a reference letter from their PhD advisors. For students who will receive an Eusflat grant, the preparation of a poster for the poster session is mandatory.

As I previously mentioned the two years of the mandate of this Board are going to an end; in Gijón the elections of a the new Board will take place. I am very grateful to all colleagues, friends of EUSFLAT who supported this Board two years ago, and for the trust they have placed in us.

These two years have constituted a beautiful journey, in which the Board has worked to maintain the quality of the Society (that was achieved by the previous Boards), and to launch new initiatives (like the School). I would like to express my gratitude to all the EUSFLAT board members for their invaluable help and support during these two years.

Last but not least, I thank all of you for your presence and your continuous support.

Looking forward to meeting you in Gijón, on behalf of the whole EUSFLAT Board,

Warm wishes

Gabriella Pasi
President of EUSFLAT

INTERVIEW

Dialogue between Etienne E. Kerre and Javier Montero

Etienne Kerre and Javier Montero surrounded by other colleagues at IFSA 2013.

JAVIER MONTERO: I think we should start this conversation acknowledging Humberto Bustince for the great job he and his team at the Public University of Navarra are doing with the EUSFLAT Magazine, which is becoming a reference between our colleagues around the whole world. It is a great honour to be invited for this conversation with you, and a challenge after the wonderful conversations between Lofti Zadeh and Ron Yager; Enric Trillas and Alejandro Sobrino; Didier Dubois and Henri Prade; Janusz Kacprzyk, Jose Luis Verdegay and Gabriella Pasi; Irina Perfilieva and Antonio di Nola; Radko Mesiar and Peter Klement; and Francesc Esteva and Lluis Godo. In these kind of situations, when someone does not how to start, we have in Spain a famous sonnet to jump into any topic: "A sonnet Violante bids me write, such a grief I hope never again to see; they say a sonnet's made of fourteen lines; Io and behold, before this line go three." (Lope de Vega, "Instant Sonnet", translated by Alix Ingber, 1995; in Spanish: "Un soneto me manda hacer Violante, que en mi vida me he visto en tal aprieto; catorce versos dicen que es soneto: burla burlando van los tres delante.") So, starting from the list of previous conversations, one idea comes to my mind, and thinking precisely about you, Etienne: it is easy now to find out how much you have done to promote the theory and applications of Fuzzy Sets from the University of Ghent. Young readers can simply surf the Web for your papers, the list of your Ph.D. students, and the amazing number of "Fuzzy" activities you have somehow launched, promoted and supported; but it is not so easy to tell what you have received back from the Fuzzy Sets community after all your efforts...

ETIENNE KERRE: First of all I agree with you to say thanks to Humberto for editing the EUSFLAT magazine. I really enjoyed the previous conversations between some pioneers of the Fuzzy Set Theory and its applications! I hope that our conversation will be appreciated too. Now concerning your question about my life with fuzzy sets, let me first say a few words about my pre-fuzzy life! I obtained a graduate in mathematics from Ghent University in Belgium in 1967. I submitted a research project to the National Institute for Research in Industry and Agriculture and I was very happy to

Etienne Kerre and Lofti Zadeh in San Franscisco.

be one of the 10% selected candidates to perform research on the diffraction of electrons by crystals. Two years and three months later I obtained the Ph.D. degree in mathematics. After my Ph.D., I became an assistant in the department of mathematical analysis at Ghent University where I became a big fan of the so-called modern mathematics based on Set Theory! During three to four years I have put all my energy into the modernization of the basic courses on mathematical analysis for undergraduates in mathematics. So my pre-fuzzy life has to do with mathematical analysis, theoretical as well as applied! Now coming back to your question: I cannot imagine my scientific as well as my social life without fuzzy sets! Already in 1974 I received a bad smelling Xerox photocopy of Lotfi's seminal paper "Fuzzy sets". At that time there was no internet, no email, only a few journals where after some time you could obtain a copy if you were lucky that your university had a subscription to the journal! The title of this paper made me very curious to see its contents because in my mind sets were very clear and crisp entities with sharp boundaries and certainly no fuzziness present! At that time my mind was indeed full of the strict mathematics as taught or exposed by the famous BOURBAKI group composed of big French mathematicians such as Jean Dieudonne, Henri Cartan, Maurice Frechet who introduced beautiful structures such as metric spaces, normed linear spaces, Banach spaces, generalized derivatives, differential calculus... They considered Cantor's Set Theory, based on binary logic, as the base of all mathematics (algebra, geometry, differential and integral calculus, etc.) and from time to time they published the so-called "fascicules", a kind of small booklets where one specific domain was developed starting from scratch. I still remember the definition of the empty set that consisted of a long list of letters and symbols from the alphabet! Beautiful mathematics but not daily workable. With this background I have read Zadeh's paper and I never got away of this enrichment of our binary models! Now Javier tell me your fuzzy starting point!

JM: Well, I think I was born to be fuzzy! I can neither imagine my life without Fuzzy Sets Theory. . . It was 1978 when Prof. Sixto Ríos, head of the Department of Statistics and Operational Research at my University and the principal person in this field by then, asked me if I wanted to start my Ph.D. studies on Arrow's Impossibility Theorem, in principle nothing to do with fuzzy sets. After one year reading books and papers on how difficult it was to keep rationality in a democratic decision system I was overwhelmed with the number of variants of the same theorem, and I went to his office and more or less I told him that I did not want to find a another similar impossibility theorem, but something really original. And then he said to me: "Try with this paper". And then he gave the paper he had in his hands: Lotfi Zadeh's 1965 seminal paper (by the way, also in a bad smelling photocopy like yours). In my Ph.D. thesis I started claiming for a fuzzy approach to all those elements in Arrow's theorem, particularly for a fuzzy rationality, and then all those fields I was interested in started to be contaminated with the fuzzy argument (among other things, I was teaching a course on Systems Reliability). I think that even my brain got definitively contaminated with fuzziness, as a very good friend of mine pointed out some time ago: "you are being binary" is for me a very serious criticism when someone assumes any excessively strict position in life. And I remember myself explaining to my kids that "If you ever see something absolutely clear, be sure you are wrong and keep thinking on your problem". In fact, the tools we use for thinking are not independent from our thoughts and feelings (as Lotfi Zadeh has said, "When the only tool you have is a hammer, everything begins to look like a nail."). You should not expect, for example, a kind discourse from people convinced that binary logic is the unique logical tool. Assuming that the world is binary is the announcement of a restrictive and stressing view of the world, where people who are not our friends must be therefore enemies, or that our acts can be simply classified as good or bad. Coming back to the field of my Ph.D. dissertation, I think that the underlying message of a logical

Javier Montero, Lotfi Zadeh and Javier Montero's wife at the ceremony of Zadeh's DHC at Oviedo (December 1, 1995).

uniqueness is terrible for a Society. Education should focus not only in acquiring knowledge but should mainly provide an attitude based upon ethical principles towards personal honesty, respect to the others and our capacity to learn, all together being conscious that the world is complex. But if ethical principles are viewed as crisp, surely we will find ourselves in a continuous sequence of contradictory situations. Our decisions should be illuminated by our principles, which are, or should be fuzzy. Negotiation abilities are much more important than bargaining abilities. Maximizing economic benefit is a dangerous outcome of a simplifying crisp culture. I am particularly worried about the consequences of the economic crisis we are currently living, not only because the direct consequences on people, but also because in some countries they are severely cutting back in future. Main decisions of a Government should pursue more health, more safety, better education and better research. This is what a democratic Society is for. Budget cuts in education and research means not only reducing our Society capacities, but furthermore important, without education and research Society loses the correct human attitude. No matter where we are living and working, we need some understanding of our surrounding world and some innovation spirit. Anyhow, I think that our position as University professors has given us an exceptionally grateful profession, don't you think?

Etienne Kerre with the winners of the best paper award at ISKE 2013, in Shenzhen.

EK: You are right, becoming a professor at the university is a very big present. It gives a lot of freedom, sometimes too much in the sense that the job is never finished, especially with respect to research, and that sometimes one forgets that there is a life besides the university. Do you know that during many years I have been teaching about 18 hours a week! The professor in charge became seriously ill and could not continue lecturing. I, as his senior assistant, was appointed to take over the lectures but there was nobody to take over my lectures on the exercises, so the work doubled. During all these years I have been working around 75 hours a week, because I didn't want to eliminate my research but I could manage it because I loved my job! Since my retirement 5 years ago I am only missing the lecturing part of the job. Luckily from time to time I am asked to give some lectures in my previous courses on fuzzy sets, for example related to fuzzy topology which was the starting point in my fuzzy

Etienne Kerre, Javier Montero and other colleagues at FLINS 2014.

research. I was responsible for the basic courses on mathematical analysis to the undergraduates in mathematics, physics and computer science. During these courses from time to time I was making some publicity for the Fuzzy Set Theory telling them for example that the binary classification in good and bad objects is a very rude one. One of my famous example is with respect to the convergence of sequences where we divide the class of sequences into convergent (good) and non-convergent (bad) ones. The real sequence (1,-2,3,-4,5,-6,...) is clearly a non-convergent sequence. Also the sequence (0.001,-0.001,0.001,-0.001,0.001,-0.001,...) is a non-convergent one. So both sequences belong to the set of bad objects with respect to being convergent while there is a big difference between them: the first one is totally hopeless (one of my professors should have called it: red-hot divergent!) while the second one is almost convergent to zero. So if convergence should be introduced as a gradual notion the second sequence should have obtained a high degree. By telling from time to time such kind of examples I could attract the attention of young people in this new theory and invite them to take one of my courses on fuzzy sets. From the very beginning I was convinced about the importance of teaching and already in 1979 I was authorized to organize an optional graduate course on the basic principles of Fuzzy Set Theory that was attended by 35 people, most of them being professors from sciences and engineering. This was an important starting point because I could convince the faculty of sciences to organize an obligatory course "Fuzzy sets and their application to computer science" for the mathematicians and computer scientists. I think that Ghent University has been a pioneer in providing such course in the regular program and it has been an example for other universities to introduce such a course in their curricula! That course was very successful and attracted excellent students who after their graduation stayed with me to prepare a Ph.D. My goodness I have been so lucky to have the opportunity to guide such excellent students! Most of them became already a professor and are continuing teaching and research on soft computing techniques! That makes me very proud!

JM: You are right, Etienne. Teaching, research and hard work, as long as they come with passion and freedom, suggest a wonderful job, difficult to defeat if you are lucky

Celebrating Prof. Da Ruan 50th birthday with his parents.

enough to gather a nice team around you. Students make us feel always young! You indeed should be proud of your students. One of the things that made me admire you - not the only one, of course- was your ability to attract such tough and brilliant students. Some of them are now prestigious colleagues. Among those students of yours, I must recognize that I still miss the energy of our friend Da Ruan. His death was the saddest moment while I was President of EUSFLAT. . . I use to say that the best way to know who you are is to look at your friends. Similarly, somehow the success of our students is the best compliment for us as professors. Still, beside our personal history, and after the first 50 years with Fuzzy Sets Theory precisely this year, our fuzzy community has been able to convince the whole scientific community that fuzzy sets can help to generate attractive alternative tools for many different problems. But do you think that we can consider that our theoretical model is basically closed?

EK: Javier if you don't mind before answering your interesting question I would like to say a few words on the way I entered into Fuzzy Set Theory. As said before I spent a few years on the modernization of the basic courses on mathematical analysis and after that I was looking for a new research domain related to mathematical analysis when I saw Zadeh's paper and some of the rather straightforward fuzzyfications of the existing mathematical structures leading to the birth of: fuzzy topology, fuzzy groups, fuzzy vector spaces, fuzzy geometries. . . . My first steps into the fuzzy world were undertaken in the domain of fuzzy topology, more precisely on the characterization of a fuzzy topology by means of a pre-assigned operation. We could easily show that a fuzzy topology in the sense of C.L. Chang could be characterized in terms of closed sets, by an interior operator, by a closure operator but not by means of neighborhood systems. For the first time I realized that while generalizing a concept or a structure one can lose some properties. At the same time this was my first fuzzy frustration: in some papers it was stated that a straightforward fuzzyfication of the crisp neighborhood system also leads to the characterization of a fuzzy topology, mentioning that the proof was straightforward! After a few years we could find a counterexample stating that this conjecture was false and we could completely solve this characterization problem by means of an adapted neighborhood concept using fuzzy points and fuzzy singletons. In this way we could completely analyze the many different generalizations of the neighborhood concept in the framework of Lattice Theory. During those years I could also experience the power of a conference to inform the participants about new topics. Every four years mathematicians gather for the International Conference on Mathematics, a very big event with thousands of participants and where also the so-called Field Medals, considered as the substitute for the missing Noble prize for mathematics, are awarded. At the ICM 1978 in Helsinki I could present during 12 minutes my first fuzzy results concerning the characterization of a fuzzy topology in one of the 36 parallel sessions. One of the listeners to my talk was Prof A. Mashhour, a famous classical topologist from Mansoura University in Egypt. For him it was the first time he heard about fuzzy sets and fuzzy topology and he went back to Egypt starting the well-known flourishing Egyptian school of fuzzy mathematics and fuzzy topology! Now, coming back to your question, of course our theoretical model is not yet finished! There are still so many domains to develop from theoretical as well as practical point of view! Especially from point of view of applications I see still many challenges for the young researchers. One of them is the further development of one of my favorite topics: fuzzy relations. Sometimes science has been described as the discovery of relations between objects in a very broad sense and in many domains one is looking for existing relationships between notions and structures, especially in medical diagnosis. Looking to the current work of my ex Ph.D. students: Peter Ottoy, Bernard De Baets, Gert de Cooman, Martine De Cock, Chris Cornelis, Guoqing Chen, Xuzhu Wang, Steven Schockaert, Mike Nachtegael, who became professor somewhere I can be proud and full of confidence with regards to the further development and application of this beautiful and powerful tool! Javier your group and Spanish institutes and universities have contributed so much to this theory, but I wonder what about the practical applications and help to industry?

A group of young fuzzy spaniards at IFSA 1985.

JM: For some reason Fuzzy Sets Theory was successfully soon expanded in Spain, with a number of researchers and groups that started to work on Fuzzy Sets. Indeed the energy of Enric Trillas played a key role, from the early beginning of Fuzzy Sets in Spain, to the creation of the Spanish Society for Fuzzy Sets (ESTYLF) and its further transformation into our European EUSFLAT Society. But I always declare that important things never happen because a single cause. . . If I go back to my memory, in the late 70t's, when I started to work on my Ph.D., there was not much research as such around me. Good professionals at the University were mainly devoted to study and teach. Only a few people published regularly in international journals. By then publishing in international journals was not really needed for the promotion within Spanish universities, and you didn't expect to get a better job outside the University even if you had proven to be able of doing research. There was not much research culture in that Society ("let others invent it" was ironically proposed by a famous Spanish philosopher one century ago). Somehow a whole generation of young researchers decided to jump into competitive research. Democracy had just been established in Spain, and despite all economic, social and political problems, the Spanish Society was willing to feel part of Europe, to end with the long isolation of Spain. This generation of researchers, as part of its Society, deeply wanted to become Europeans. We were willing to travel, to see what was going on in other countries, to learn from them, to start new adventures. Perhaps I have an excessively romantic memory of those days, but I think that such a creative energy that came with democracy brought to the universities in Spain an absolute need for new projects and new ideas. But these new projects had to be launched taking into account the poor scientific infrastructures we had then (when Spain joined the European Union January 1, 1986, things started to change). Our main resource was our own creativeness. In my case, my Ph.D. advisor retired immediately after I finished my Ph.D. in 1982. I paid from my own pocket my first national and international conferences, till I got my first Grant, several years later. Nobody told me ever the convenience of submitting a paper to an international journal. Nobody helped me to write my first papers. Nobody explained me how to apply for a national science foundation project. . . I have a wonderful memory of the 1985 IFSA conference in Palma de Mallorca, where I met Lotfi Zadeh and all the important fuzzy "popes", including Enric Trillas and all those, now not so young, Spanish colleagues whose friendship I still cherish. I think that Fuzzy Sets appeared in Spain as a brand new field where many interesting things could be done, a new theory still to be developed with lots of suggestive applications. A perfect topic to start a new life! The collaboration with industry started to grow exponentially, and the creation in 2005 of the Soft Computing Centre in Asturias (where the IFSA 2015

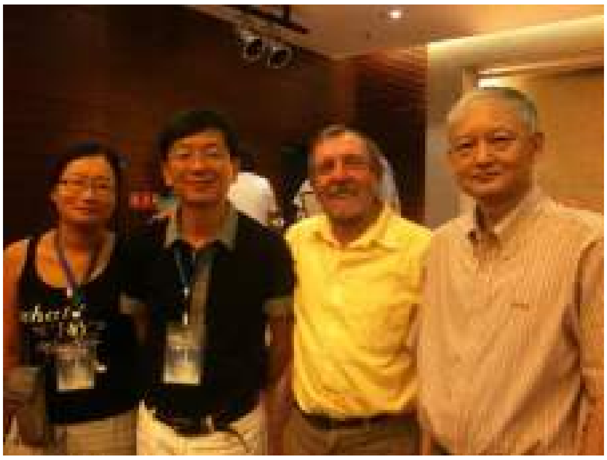
A moment at FLINS 2014.

A group of fuzzy researchers at the ceremony of Enric Trillas's DHC at Pamplona in 2014.

conference will be held) is a major achievement. The economic crisis and those policies that do not recognize the direct and indirect impact that research outcomes always brings back to industry and the whole Society is jeopardizing part of the good things achieved in Spain during the last 30 years. In Spain, as in many other countries, we are particularly suffering from poor institutional support to young researchers. I hope this nightmare will end before long, since a Society that does not promote research soon loses the adventure spirit that we need to build a better future.

EK: I also vividly remember the first IFSA congress in Las Palmas in 1985 where I presented a paper on fuzzy information retrieval and databases, invited by the other pioneer Hans Zimmermann! I cannot remember if we met there for the first time but I still have the flag with the welcoming address that hung at the entrance of the campus where the congress took place: it was some kind of hand painted piece of linen material, too nice to disappear in the dust-bin! As you said it was not that easy to attend conferences from financial point of view. I remember that those days we could get from the Faculty of Sciences every two years ten thousand Belgian francs (this is less than 250 EUR!) for attending a conference, in most cases it was only enough money to pay for the registration. But because of my lecturing on fuzzy sets and the popularity of the practical applications in Japan, soon after the first IFSA congress I was contacted by former master students who were employed in industrial companies to search the possibilities of applying Fuzzy Set Theory in their plant. For example in 1987 I was contacted by Dow Chemical Company who had a branch in the south of The Netherlands, close to the Belgian border. They had a problem with a large amount of data to control in real time. Their best experts were not always available and they wanted to build a system that could help whenever some component failed and has started to fail other components and at the end shut down the whole system. So we have built an IDR, Intelligent Data Reducer based on Fuzzy Set Theory, more precisely on Fuzzy Reliability Theory where we allowed degrees of relevance. During three years I got money to pay young researchers such as Gert de Cooman and Bernard De Baets and to attend conferences and buy computers! Other companies for which we have done applied research and lectured the basic principles of Fuzzy Set Theory are Agfa Gevaert where we worked on the selection of patents, Lastekon NV where we developed a semi-automatic welding machine and Alcatel Bell, where

I gave a lot of lectures. Now I also want to come back to Da Ruan! In 1987 a bilateral cooperation between Ghent University and the prestigious Fudan University in Shanghai (one of the four top universities in China) has started where every year 2 students from Ghent could go and study at Fudan University and vice versa. That year Da Ruan got after a strong selection and based on his excellent master results such a grant and it was asked to the Faculty of Sciences if someone was willing to guide this guy! In this way Da became my first Chinese Ph.D. student! After his Ph.D. in 1990 he went to the Belgian Nuclear Research Center where he performed theoretical as well as applied fuzzy research in the domain of nuclear research. After his Ph.D. and till his sudden death in 2011 we stayed in close contact almost every week by phone, email or physically! He was one of my best friends and I am very grateful to you Javier that I got a lot of support in erecting a steering committee in order to continue the work of Da with respect to the successful FLINS and ISKE conferences. I am still missing the friendship of Da very much!



Etienne Kerre and thre of his PhD students at FLINS 2010.

JM: The fuzzy community has been always a friendly community, most probably influenced by the personality of Lotfi Zadeh. Fuzzy community should never lose that sincere scientific spirit of collaboration with as many colleagues as possible, even far away from our specific field. It is very important to enjoy our work through the inspiration of new ideas. I use to recommend my students that they should not have just one single research interest, and that part of their time should be devoted to read and attend conferences about topics in principle not related to their research. You should not fish always in the same river if you are looking for a different kind of fish. Moreover, despite all the great work done within the fuzzy field, there are key issues still to be addressed. For example, I think we do not have a good formalization of fuzzy experiments, how they can be run and managed. Long time ago a student of mine asked me, once a long course on Statistics was finished, what was the most important thing of Statistics, and I remember I answered that the most important things in Statistics were those that refer to how to observe reality (the design of an experiment), and how results should be presented so users can understand data. In between observation and nice graphics there was simply mathematics, I said. With a good data set many interesting things can be done, but nothing interesting can be obtained from a bad data set. The concept of experiment is precisely formalized in the crisp framework. A crisp experiment needs crisp results, crisp events and specific conditions in order to guarantee and validate results. But I think we have not still been able to fully agree on how fuzziness connects with reality by means of fuzzy experiments. Kind of surprising since managing fuzzy information is precisely the basis of the great success of human being. Our capability to represent an indeed complex reality in terms of compact, robust but flexible (fuzzy in nature) concepts, is in my opinion deeply related to the creation of languages, and sharing a language has been essential for social learning instead of learning as individuals, simply from our surviving genes and our closest community. I have to acknowledge that my early research was focused on decision making, but when I read those studies in Psychology and Neurology about how human decision making is made, I started to believe that it was misleading to pursue a formalization of rational decision making. Rationality can be found in our argumentation, but what we do at the end is strongly influenced by too many things, too often impossible to control (emotions also play a key role). The only thing we can assure is certain consistence between our rational arguments and our move. And I strongly believe that our main decisions are likely to be fuzzy: most of our important decisions are strategic decisions about our life, where alternatives and consequences are poorly defined most of the time. It now comes to my mind that in a FLINS conference I told a personal experience, when I decided to help a homeless I saw at the end of a long street while in Berkeley, and how while walking I was not able to complete my analysis about alternatives and consequences because nothing was ever clear enough, and how I reached the homeless without any decision, so in the very last moment I picked out a random coin from my pocket. How can anybody infer from that coin I gave to the homeless that I had simply decided to be good that day? Our acts too often seem to be a consequence of strategic decisions through a chaotic process. Instead of telling people which one is the optimal decision we should help them understand their problem through a good analytic approach, developing decision aid tools aiming for a comprehensible output. . . and then allow them to take their own risk and responsible decisions. Anyhow, I think that the main ideas of my research were more or less contained in my Ph.D. thesis (some time ago

A moment of the social activity of FLINS 2012 in Istambul.

I was told that most mathematicians have a unique idea in their life, and that this idea always pops up when they are young, that's why Field Medals cannot be awarded to senior researchers). I liked Arrow's approach to group decision making, based upon ethical principles rather than any solution based upon Game Theory, but principles should never be strict and they are always subject to conflict. The key issue for me became how reality should be described (models for fuzzy classification), how complex information should be presented (for example, image analysis should help to develop some kind of descriptive statistics for fuzziness: we need to learn how fuzzy classes should be painted), how to measure the quality of a fuzzy classification and how to measure consistency (which in my mind was from the beginning conceived as a fuzzy property: still now I see some standard fuzzy definitions as dangerously crisp). In my Ph.D. thesis I also started to focus on how things can be effectively reckoned (for example, avoiding the strong associative condition which assumes the existence of a unique operator for sequential aggregations, which will later on lead to a more general but operational recursive approach). At the end, we should keep stressing that we cannot miss relevant elements of reality in our mathematical modeling. In this sense, two of my first papers in Fuzzy Sets and Systems journal, also based on my Ph.D. thesis, pursued to justify that the simplest representation of fuzziness requires several simultaneous connectives, to be able to answer several questions at different extensive and comprehensive levels. For example, to what extent John or Paul are tall, to what extent John is tall or very tall, and to what extent John is tall or blonde. Not taking into account the relationship between pixels in an image or between categories in a classification, for example, may produce misleading results. In particular, a key argument in my research has been to stress the underlying structure of systems...We even published a joint paper in Fuzzy Reliability Theory, don't you remember? Having the opportunity of working with you and other colleagues and students has been great. I am fully convinced of the advantages of team work. Lots of wonderful moments sharing ideas with colleagues come to my mind. Many of those colleagues are now close friends. I have no words to thank all the sincere friendships I have found while working!

Enjoying THE dinner at IFSA 2013 in Edmonton.

EK: Indeed Javier, we got a lot of friendship from the fuzzy community. Every time we attend a conference we are

A group of mature fuzzy spaniards at the ceremony of Lotfi Zadeh's DHC at Oviedo (December 1, 1995).

warmly welcomed by our colleagues, especially the old ones that became real friends, as we can see around new year when one email after the other is coming in wishing the best for the coming new year. Surely Lotfi is a binding factor, he has shown the way to be friendly and open minded! I very much appreciate the sentence finishing his talks: please don't hesitate to disagree! His kindness and concern is legendary . I remember when we had breakfast together during the Fuzzy Days (my goodness I miss this series of conferences -at least 10- organized by Bernd Reusch in Dortmund, Germany) that Lofti advised me: eat your breakfast yourself, share your lunch with your friends and give your dinner to your enemies! Even during breakfast he advised me to take some extra piece of fruit because later on, he said, there will be no more fruit offered! Another good memory, my goodness I have so many from my fuzzy life, is the dinner during the IFSA conference in Edmonton when we were sitting on a round table with about 20 people listening to the vivid life stories of our friend Burhan Turksen! Unforgettable! Of course I think we have more than one joint publication. Do you know I still have the poster announcing your first talk in my department on March 21, 1994 about Consistency Measures of Fuzzy Preferences? I agree with you that it is important to stimulate our students to become familiar with other domains. In this respect I regularly organized in my department one hour talks on differing subjects by eminent researchers such as you, Radko Mesiar on compensatory operators, Ania Radzikowska on fuzzy rough sets, Hung Tung Nguyen on random sets, Gabriella Pasi on a fuzzy object-oriented data model, Laszlo Koczy on fuzzy controllers, Huang Chongfu on the prediction of natural disasters such as earthquakes, Sandor Jenei on non-classical triangular norms, Sergei Orlovski on describing natural concepts , Janos Fodor on uninorms, Henk Heijmans on mathematical morphology, Peter Klement on fuzzy logic in Linz Laboratorium, Guoqing Chen on mining association rules, Maciej Wygralak on fuzzy cardinalities, Yuji Yoshida on fuzzy decision making, Madan Gupta on fuzzy-neural computing systems, Peter Walley on belief functions, Pengfei Shi on imaging engineering in China, John Dockery on virtual reality and the disabled -an exercise in tailoring a fuzzy world-, Vilem Novak on mathematical fuzzy logic, Patrik Eklund on metric image spaces applied to texture analysis in mammography, Krassimir Atanassov on intuitionistic fuzzy sets and Jie Lu on recent developments on E-service intelligence. It is already a long list , there are more and I apologize to those I forgot, but this list shows that informing students on new topics can lead to new interests and new developments. Such talks have inspired several of my students to start research leading to a Ph.D.! Let me end with some thoughts about the future of our research domain. First of all the fuzzy community should agree about the definitions of the basic concepts (for example a fuzzy number, a fuzzy topology...) and determine the essentials that should be known by newcomers in the domain and present to them in affordable textbooks. I know there are already several textbooks but I am not completely satisfied with any of them because they are mostly written from the personal background of the author(s). Another point of attention concerns the fast growing number of journals on soft computing. For a long time there have been only a few journals that accepted papers on fuzzy sets. Nowadays there are more than 25 journals with fuzzy in the title, this is too much and will at the end negatively influence the quality of the accepted papers. Regularly I hear colleagues complaining about the huge number of papers to referee! I myself have been referee for more than 70 journals with around 80 papers per year! Many times I experienced that I wrote a negative report on some submission that some months later I saw published in another journal! A similar warning can be made about the increasing number of conferences in our domain. Let's keep watchful concerning the quality of our journals and conferences! To conclude I want to say, my dear friend Javier, that I am very happy that I discovered Zadeh's paper "Fuzzy Sets". It enjoyed my life and I sincerely hope that more and more people, especially mathematicians, will become aware of the power of this beautiful gradual theory and start to further develop it in all possible domains in order to make our world better!

Etienne E. Kerre
was born in Zele, Belgium on May 8, 1945. He obtained his M.Sc. degree in Mathematics in 1967 and his Ph.D. in Mathematics in 1970 from Ghent University. Since 1984, he has been a lector, and since 1991, a full professor at Ghent University.In 2010 he became a retired professor. He is a referee for more than 80 international scientific journals, and also a member of the editorial board of international journals and conferences on fuzzy set theory. He was an honorary chairman at various international conferences. In 1976, he founded the Fuzziness and Uncertainty Modeling Research Unit (FUM) and since then his research has been focused on the modeling of fuzziness and uncertainty, and has resulted in a great number of contributions in fuzzy set theory and its various generalizations. Especially the theories of fuzzy relational calculus and of fuzzy mathematical structures owe a very great deal of him. Over the years he has also been a promotor of 30 Ph.D's on fuzzy set theory. His current research interests include fuzzy and intuitionistic fuzzy relations, fuzzy topology, and fuzzy image processing. He has authored or co-authored 25 books, and more than 500 papers appeared in international refereed journals and proceedings.

Javier Montero
Javier Montero is Full Professor at Complutense University of Madrid. He has been leading research projects since 1987 and has published more than 100 contributions in JCR journals, mainly devoted to the theory and applications of fuzzy sets. He has served the fuzzy community as President of the European Society for Fuzzy Logic and Technologies (EUSFLAT) and as Vicepresident of the International Fuzzy Systems Association (IFSA). He has been also Dean of the Faculty of Mathematics and Vicerrector at his University.

SCIENTIFIC REPORT

A fuzzy day with Javier Montero: A personal vision about some of his scientific contributions in the events of a usual day

Daniel Gómez
Facultad de Estudios Estadísticos, Universidad Complutense, Avenida Puerta de Hierro s/n 28040, Madrid, Spain
E-mail: dagomez@estad.ucm.es

1. Introduction

After obtaining my degree in Mathematics in 1999, I had the pleasure to meet professor Javier Montero. I was starting my pre-doctoral courses when I met him. I had already decided most of the courses to attend except one and I was vacillating between *Generalized Markov Processes* and *Decision Making in fuzzy environment*. Finally I took the second option, probably because the words decision making in the title attracted my attention more than Markov processes. Now I can say that probably this decision was one of my best choices from a professional point of view.

In this paper, I will try to write about Professor Javier Montero, some of his important achievements in science (from my point of view) and I will try to explain some of his characteristics as a researcher and as project leader. It is important to note that this is only my personal opinion and my reduced vision of his main achievements.

Starting from the beginning, there is one characteristic that is inherent and perfectly defines Javier Montero as a researcher. I think that this characteristic clearly protrude over others. Javier Montero is always questioning the things that are assumed as certain by most of the researchers. When I think in Javier as a researcher, I have the picture in my mind of other classic romantic researchers trying to convince the rest that they are wrong (now I am imaging Cristobal Colón (Christopher Columbus) speaking with others about the fact that after *Finisterre cape* there is not a dragon or a monster that will eat the ship that go through). Also when I see Javier Montero writing the projects in which he has been the leader during the last 30 years I really see someone as Cristobal Colón asking for money to Catholic Kings of Spain for his new expedition.

I remember the first lecture that I received from Javier as if it was yesterday. Probability theory was for me a closed and seamless theory. Javier was the first of my teachers to point out the main debilities and wrong hypothesis of probability theory. Javier always said *Science has a lot of tricks and probability theory more*. For example, the concept-idea of *uncertainty* and his different meanings (see following picture). It was something surprising for me to realize the existence of other types of uncertainty in addition to the probabilistic one. Or also the concept of statistical independence, always present as an essential hypothesis in most of the models of statistical inference or data analysis.

Photograph in which Javier shows me the difference perception of reality of fuzzy sets and probability.

Probably, this characteristic of Javier Montero was (in addition to all of his human abilities) the main reason why I decided to make my PhD thesis with him. Since then, I have been closely working with Javier during the last 15 years and it is clear that this relationship has been more beneficial for me than for him. Now, I am pleasant to have the opportunity to say thanks in this paper for all the things that he has given to me.

Just to conclude this awkward introduction, I have to say that this work is devoted to bring an idea about some of the scientific ideas of Javier Montero in the events of a usual day in his life. Starting with the alarm in the clock of his bedroom and finishing when he returns back from the office.

2. A normal weekday?

First at all, I have to say that there is not a common weekday in Javier's life since each day is different. A standard professor in Spain divides his work in three important pillars: Teaching, Researching and Management. Since I met him 15 years ago, Javier has put different intensities in these three areas depending on his professional period. For example, from 2013 to February 2015 Javier was Vicerector of the Complutense University of Madrid (that is the biggest university in Spain with more than 100.000 employments among administrative personal and professors). So it is clear that during that period a day in Javier's life was very different from now. Taking into account this disparity, I have decided to talk about a weekday (that is not normal) in the present days in which Javier dedicates his time to play with new research ideas alone or in friends meetings, reading, connecting ideas and people, teaching (from the beginning of uncertainty to new degree students until the last advances and news in science to post-doctoral students), and making management. Fortunately, for us, in this new period we can see Javier more and we enjoy discussing with him new ideas, taking the advantage of his way of pushing us to give all the best.

Wake up !!!!

Beep, Beep, Beep... Javier hears the alarm of his clock. His vision is fuzzy and he can't distinguish the numbers of his clock. Everything is very fuzzy in the morning. It should be 7 a.m. more or less. Now its time to wake up. Marise (Montero's wife) tells him (gently) that he has to switch off the alarm. Otherwise everybody in the house will wake up. Finally, Javier wakes up and he goes to the kitchen thinking about how he will organize the day. Javier's body is asking for an orange juice, a coffee with milk and some cookies. Nevertheless, Javier is in a diet and he does not carry for this menu.

As I said previously, one of the main characteristics of Javier Montero as a researcher is that he is not carried away by the easier, by the common. Analyzing or reading the papers that Javier Montero has published, we can see this factor as a common characteristic. As an example, I will told about my Phd thesis (supervised by Javier Montero and Javier Yáñez). This was my *wake up* as a researcher. Multicriteria decision making problems was one of my favorite issues during that time. In these problems, a usual assumption is that there exist some decision alternatives, some criteria and the decision maker is able to order the alternatives based on each criteria or to quantify a value/score of each alternative in each criteria. Although these assumptions are usually accepted, they are too much for Javier Montero. In 2001, Javier put me some examples to show me that sometimes it is not possible to know your own criteria (even if you are able to take decisions), other times the idea of criteria is very general and you cannot order the alternatives based on this criteria. Taking into account all these ideas, the searching of the criteria that permits a good representation of a preference data set and a fuzzy preference data set has been one of the topics of Javier Montero in order to understand the decision making problem ([66, 33, 34, 24]).

The dimension concept for preference relations permits to understand better the decision maker problem, to search his criteria and also different types of rationalities. Given a partially ordered set (or poset) R (that could model a classical preference relation of a decision making problem), the dimension of P, $Dim(P)$, is the minimal number of linear orders ($\{L_1, \ldots, L_{Dim(P)}\}$) that permits to build $P = \bigcap_{i=1}^{Dim(P)} L_i$. The linear orders L_i model the classical criteria that permits to explain the preferences of the Decision Maker. If the preference relation R is not a partially ordered set, the dimension of R ($dim(R)$) was defined in [66, 33, 34] as the minimal number of linear orders in such a way that $P = \bigcup_i \bigcap_j L_{ij}$. This definition permits to explain the lack of transitivity, rationality or inconsistency that appears when the preference relation is not a partially ordered set. Also, this definition permits to extend the idea of dimension (see [66, 33, 34, 24]) to fuzzy preference relations. Here the idea of representation permits the understanding of a fuzzy preference relation, rather than some numbers in a matrix.

The analysis of the preferences in decision making, multicriteria decision making or group decision problems, has been one of the main challenges of Javier's research ([14, 49, 50]). During that period, most of the preference relations analysis (in a fuzzy or crisp framework) did not take into account all the structure that appears in Decision makers mind (strict preference, weak preference, incomparability and indifference). This omission, has strong consequences, as the fact that the classical Orlosky intensities are not always compatible with this structure ([44]). The idea of transitivity of a preference relation appears also in consonance with the analysis of the preferences. In 1986, he published his first paper on that topic on FSS [38], in which he proposed a method to find (for a given fuzzy relation) the closest rational and transitive fuzzy relation according to the preferences of the DM. From this paper until now ([14, 40, 66, 66, 33, 34, 15, 50, 22]) he has dedicated many efforts to deal with the idea of transitivity of a fuzzy relation, the notion of rationality and coherence in preferences, and his consequences to the study of preference relations, decision making problems, group decision making problems and also multicriteria decision making problems.

Arriving his second home

During the breakfast in Montero's house is time to decide the allocation of task between Marise and Javier. One of them will have to take his three children to school. The discussion and organization about the pros and cons takes all the breakfast. But at the end and without any clear consensus Marise takes the keys of the big car to carry their children to the school. During the discussion, she realizes that today she has to go against the traffic jam because she has a meeting in other place. Thus Javier takes the small car to go to the university and to *the Faculty of Mathematics, his second home.*

From Javier's point of view, complex decisions have a long process that at the end finishes with a crisp decision, but the previous process is usually fuzzy. Javier always says, *Crisp acts but Fuzzy decisions*. One has to differentiate between a fuzzy decision as for example: *to be good* or *to help somebody*, *to buy a non expensive car* and the crisp decision that is the final output of the decision process. In this sense, Javier (see for example [39, 1, 38, 45, 16, 17, 8, 52, 62, 19, 20, 21, 67]) has developed many fuzzy models that could be a fundamental tool in the decision processes, instead of referring to the final output of a decision, which is a crisp act. In fact, while probability theory can properly model crisp acts, this is not the case for fuzzy decisions. This is the reason why we have to carefully take into account the information that we receiv in a decision making problem, since we only see the final output of a long mental process. Probably this is the reason why the classical ideas about consistency, rationality, and transitivity (as we will see below) should be relaxed.

In [52], I can read his thoughts:

...human in general manage poorly defined arguments and alternatives, and what we usually call a decision is always ill defined, although we still expect they should produce consistent acts as their consequence. These acts, still being consistent with the previous fuzzy decision, are somewhat unpredictable, being extremely dependant on the specific circumstances at the moment an act is required by decision makers. Therefore, in many cases pursuing consistency of these subsequent crisp acts may be misleading. The main issue should be in principle to check consistency between those acts and the true decision behind them, usually poorly formalized. But this objective may sometimes be unrealistic, since there may be few chances to repeat the experience. We should also focus our attention on the arguments that led us to such a poorly defined decision. This should be a relevant role of fuzziness in decision making, viewed as a decision support problem (acts are supported by a fuzzy decision which is supported in fuzzy arguments).

The ideas of consistency in decision making models, that have had a significative impact in the developing of preference models during the last years, appear as a natural consequence of focusing the attention in the decision models. Also the idea of consistency will follow to Javier in other research topics as: aggregation processes (see [64, 65, 63, 30]), classification problems (see [3, 4, 1, 5, 25, 59, 60]) and image analysis ([27, 28, 7].)

Discussing in the cafeteria

It's 8:30 in the morning, and Javier has answered some urgent emails. The door of his office sounds (knock, knock, knock...). Its Javier Yáñez, one of his oldest friends of the faculty, for more than 30 years. Javier Yáñez wants to talk with him since there are some questions that have to be solved. At the same time, other colleagues also enter in the office to talk with him. Javier decide to go with everybody to the cafeteria since the problems are related. During the corridor that goes to cafeteria Javier meets with other two or three professors that want to talk with him. It's time to aggregate the information that he receives to give instructions to his people. He never slows down!

Javier Montero with his friend and colleague Javier Yáñez in 1978 and 2008.

Aggregation is probably the topic that is in the intersection of his different research. The idea of aggregation appears in most of the research topics that he has been working out, since aggregation represents a fundamental tool in: classification, group decision making, multicriteria problems, image analysis and optimization, among others (see [7]). It is clear that aggregation is one of the most important tools in science, and in particular in knowledge based systems. In general, we can say that aggregation has the aim of using different pieces of information to come to a conclusion, a decision and/or a classification. The first work of Javier Montero in the field of aggregation processes [37] can explain his thoughts and ideas that he has followed until now. In this first work, published in 1985 in Fuzzy Sets and Systems under the title *A note on Fung-Fu's theorem*, he realized that the axiomatic approach of an aggregation process in a decision making framework proposed by Fung and Fu in 1975 [35] was too restrictive for a fuzzy environment. Aggregation operators have been a widely topic in information systems and soft computing. Nevertheless, many assumptions have been imposed when they are applied to real problems. How to deal and how to relax these classical assumptions have been ones of the challenges in Javier Montero's works during the last 30 years. Here we put some examples (not all of them because the number is huge !!!):

- **Unicity.** A family of aggregation rules or global aggregation process usually assumes that there exists only one aggregation operator in the aggregation process.

It has been lately made very clear that aggregation processes can not be based upon a unique binary operator. Global aggregation operators have been therefore introduced as families of aggregation operators $\{T_n\}_n$, being each one of these T_n the n-ary operator actually amalgamating information whenever the number of items to be aggregated is n. Of course, we should not be accepting that aggregation operators can not evolve in time or that they simply remain constant along the complete aggregation process [1, 14, 13, 18, 16, 17]. Although it is clear that some mathematical restrictions have to be introduced (in order to assure an appropriate meaning, consistency and key mathematical capabilities), Javier can not accept, for example, only one intersection operator to represent the intersection between two fuzzy sets during the whole classification process.

- **Associativity.** Another classical mathematical property, in which it is assumed in some way that $\phi(\phi(x,y),z) = \phi(x,\phi(y,z))$. Although this property could be viewed as a necessary restriction again it is very restrictive for Javier and not desirable in many situations [48, 1]. Associativity allows recursive application of the same binary connective in such a way that memorizing only one binary connective is enough. Nevertheless, one of his favorite aggregation operators (OWA operators) [9] is not associative and Javier can not accept that the applications of OWA operators require a previously fixed dimension. For example, there are many real life decision processes that require at different times the aggregation of inputs of different dimensions. Connective rules have to be defined before knowing such dimensions.

- **Commutativity.** Commutativity is a classical mathematical property, in which it is assumed that aggregation will be invariant with respect to permutation. From Javier's point of view, this property could imply a severe restriction, since in many real applications the decision maker wants to keep track on how data were obtained, at least to be able to locate them in time, place and other circumstances. Assuming that aggregation does not depend on ordering implies that the result is not being affected neither by the time in which data are being produced nor by the time data take to arrive to the decision maker, and it is also suggesting that the result may not depend on key circumstances surrounding data that most probably should have been included as data themselves. Data are not just numbers (see for example [45, 23, 1]).

- **Recursivity.** Javier proposed in many works (see [3, 4, 13, 18, 1, 23]) a more general idea of recursivity. As it happens with other classical properties, classical recursivity definition is too restrictive and it is difficult to apply in real applications. The idea inherent in all of these papers is that the family of aggregation operators or the idea of aggregation rule should be operative and allows to build n-dimensional aggregation operators by means of aggregation operators of lower dimension. As a consequence, it is shown that if we impose the aggregation to be strictly increasing, then we can easily conclude that our aggregation is based upon weighted sums. Indeed, this is an extremely important result, with obvious consequences in any aggregation procedure. This is a quite standard situation in Multicriteria and Group Decision making, Classification and Time Series, for example. Such a basically additive representation can be then assured, avoiding the restrictive associative approach which is based upon a unique binary operator.

- **Conjunctions and Disjunctions.** Since there is only one logical structure based upon the binary $\{0,1\}$ valuation space, one may be tempted to assume that every aggregation procedure within this context should be based either upon the above (crisp) conjunction or the above (crisp) disjunction. In addition, once the space of degrees of verification is extended to the unit interval, of course we should be expecting that both conjunction and disjunction can be modeled in many different alternative ways ([10, 11, 8, 32]).

- **Idempotency.** From Javier's perspective, this property can be relaxed and even eliminated in some contexts, due to some kind attraction behavior: a high degree, if repeated, may suggest in many contexts a higher aggregated degree, and a low degree, if repeated, may suggest a very low aggregated degree.

- **Consistency and stability.** A family of aggregation operators, has been classically defined as rules, from which it is possible to deal with the information reaching to us; its dimension not being previously fixed. This is important because the number of items that has to be aggregated is not always the same, since in practice it is common that some information can get lost, be deleted or added, and each time a cardinality change occurs a new aggregation operator is needed. Taking this into account, Javier always points out that this notion of family of aggregation operators ($\{\phi_n,\ n \geq 2\}$) should present some consistency and some stability between its members. In [63, 64, 65, 30], Javier presented some properties related to the family of operators in a global way. The concept of stability (and other related concepts) for a family of aggregation operators are defined in order to establish some properties for the family of aggregation operators to be consistent. In particular, the stability property for a family of aggregation operators tries to force a family to have a stable/continuous definition in the sense that the aggregation of $n-1$ items should be similar to the aggregation of n items if the last item is close to the aggregation of the previous $n-1$.

First lecture for degree students

It's ten o clock. It's time to give the first lecture of the day. A course of probability theory for first year students. Javier proposes some problems for students and encourages them to think about the models that they have seen previous days. It is time to reflect on all the problems that can be solved by classical *models and also on those who do not. It's time to be rational...*

The idea of rationality has been in Javier's mind since he started to research in his Phd Thesis. Some of the questions that he has been trying to answer (at least in a particular framework) are related with the questions: When a person is rational? or When its preferences are rational? or When its acts are rational?. In the analysis of crisp preference relations there exist some ideas of these concepts. How to extend these ideas of rationality or assumptions into a fuzzy framework has been one of his favorite research topic (see for example [39, 42, 41, 12, 48]). Since fuzzy preference relations formalize intensity of individual preferences over fixed sets of alternatives, in [39, 42, 16, 17, 22] he gave an axiomatic basis for defining the concept of fuzzy rationality. Specifically, he established a collection of desirable conditions that any fuzzy rationality measure should satisfy. Rationality is based on the concept of *fuzzy acyclity*. He presented the rationality of fuzzy preferences as a fuzzy property of fuzzy preferences. Also, he proved that several rationality measures can be aggregated into a global rationality measure. These concepts were also applied to multicriteria decision making problems and decision group problems in a fuzzy environment. From the beginning until now these ideas appear partially in other contexts as aggregation process (related with the idea of consistency), representation decision making problems or classification problems.

Researching morning meeting

It's 12:00 and the team of Javier Montero is ready to start the meeting. Javier however still does not arrive. He will surely have been held up by some students or staff in the hall. Finally, he arrives at 12:15 and we start the meeting. First, the most urgent things: some papers that need revision and some special issues or congress with a close due-date. Also hurry the drafting and all the paperwork that the team has to prepare for the call for a Spanish project. Now it is time to discuss about one research topic. One member of the team is going to talk about the state of his research: the theme for today is image analysis and fuzzy classification.

Research Meeting of FuzzyMAD 2014.

Fuzzy classification is another field in which Javier Montero has made some contributions. Classification appears later in his interests than other disciplines as decision making, aggregation process, logic, reliability systems or fuzzy sets (in [3], we can see his first publication on that topic). Some years ago, I asked Javier what was the reason why he decided to make research in classification, after dedicating many years and efforts to decision making. He told me that after many years he had realized that to make a good decision and understand any decision making problem, you first have to make a good classification of the set of alternatives. He finished the conversation with the following quote that I will try to translate: *Dani, you have to realize that decision making is just a particular case of a classification problem where you have some classes: good alternatives, not as good, and bad ones. A good decision process is to be able to make this classification, since in most of the real situations, the final decision is made randomly (to say something), in a not rational way and it is very difficult to analyze and explain by the observers.*

A standard crisp classification is a partition of the set of objects. In a fuzzy classification, the equivalent concept is the concept of fuzzy partition defined by Ruspini. But again, swimming against the common, Javier thinks that this vision of fuzzy classification is not always desirable (at least in the first steps of a classification process). In the Ruspini approach, each membership function can be evaluated by itself, without taking into account the remaining classes. Nevertheless, most users will find serious difficulties in assigning degrees of membership to one class without taking into consideration the remaining possibilities for classification. Classification procedures usually require a previous look at all available classes. Hence, classification methods are in general highly dependent on the family of classes the user is forced to consider (even in a crisp context, users frequently have a look at all possible choices before choosing a particular class for a given object). Given a universe X, and a set of classes $C_1, \ldots, C_k$ with memebership functions $\mu_{C_1}, \ldots, \mu_{C_k}$. These fuzzy classes are a Ruspini partition if $\sum_{i=1,k} \mu_{C_i}(x) = 1,\ \forall x \in X$.

From Javier's point of view, Ruspini partition seems a desirable situation, since every object seems to be explained, in some way, with the minimum amount of information. But again Javier thinks that this definition shouldn't be the only way to represent a fuzzy classification since it is too restrictive for the fuzzy world ([1, 2, 3, 4, 5, 25, 31, 32, 29]). Usually, the fuzzy classes under consideration do not verify the conditions of such a Ruspini fuzzy partition (for example, a human decision maker will not meet those requirements if not artificially forced). From Javier's point of view, a Ruspini partition is only obtained in a fuzzy classification after a long learning process in which the classes are modified (as happens in statistics with the factors in multivariate analysis). Again the idea of Javier appears that the most interesting think is the learning process until you reach the final output. Ruspini conditions should not be satisfied in the first process and could not be satisfied also the in last iteration if the classes that you want and you have are not orthogonal. These ideas and impronta appears in all of his works of image analysis and classification problems applied to remote sensing [1, 2, 3, 4, 5, 25, 27, 28, 29]. In [2], Javier presented some desirable properties for a classification process. It is the first time in which the concepts of relevancy, redundancy (or overlap) and covering for a fuzzy classification systems appears.

Following with these ideas and the importance of the process rather than the final output in a classification process (as also happens in a decision making problem), he started to think that the information that should be showed to the decision maker or user of a classification system is the evolution of how these classes are formed until the last step. As he always saiy

Sometimes a hierarchical clustering approach seems more appropriate, since the overall splitting process can be visualized, bringing specific advantages. The knowledge of hierarchical structure can be used to predict missing connections in a more realistic way. The main advantage of this approach is that it can show an evolution (in terms of a dendrogram) of how the pixels are joined (in agglomerative methods) or split (in divisive methods), from the beginning of the process to the final step. Although hierarchical clustering methods may bring more computational problems, they have an important advantage (especially in social network analysis) because the results are more informative than those given by partitional or non-hierarchical clustering algorithms, which only provide a final picture of the process.

During these last years he deeply studied and developed the concepts of hierarchical classification (as a learning process), hierarchical image segmentation and fuzzy image segmentation [31, 25, 27].

Lunch

Lunch time in Madrid during FLINS 2008 congress.

Now, it's time to eat. Javier has finished his research meeting and taking into account the hour, it does not merits to go up to the office and then back down in a few minutes. So, he decides to go down to the cafeteria, sitting on the 'patera' table and to start to eat while he waits for his friends to arrive. The lunch time for the job-friends of Javier was always a problem during years, since the lunch time never came like everyone. For example, if someone had finished something and wanted to take a rest, others were in the middle of a process and wanted to delay a little longer. For this reason, Javier's group reached an agreement or consensus related with the lunch time and the place. They will always eat at the same table and in the same time (2 p.m. more ore less), in such a way that when someone wants to eat, he can go down and feel free to eat with whoever is at that time. In the same way when you have finished the lunch, you can go to your office although your friends have not finished eating yet.

Another topic in Javier Montero's life as a researcher, has been the problems related with decision group making, consensus and voting problems. Javier has published many papers in the framework of decision making models, preferences and also in decision group problems (see for example [46, 49, 33, 34, 19, 20, 21, 60, 61]) . In addition to the visualization and representation problem (previously mentioned), again I have to remark the importance that Javier gives to the decision processes, rather than the final decision. Aggregating the opinion of people is a crucial problem in every society.

In order to understand the impact of Javier's research in group decision making, we have to come back to his Phd Thesis. One of the first research papers that Javier read was related to Arrow's theorem [6]. In Arrow's work, it is showed the impossibility of a rational procedure in group decision making. Nevertheless, as always happen with Javier, he did not accept this dramatic solution and found an escape. Javier realized that in Arrow's model there are many unrealistic assumptions, hypothesis and objectives (some of them not explicit in the theorem) in its underlying binary logic (a crisp definition is implied in preferences, consistency, consensus and every concept or piece of information). Now the question was how to present a model that permits a possible solution for one of the main social choice paradoxes. At the same time, in the office of Javier, he found a paper published in 1965 by L. Zadeh about a new concept that he had never heard about: fuzzy sets. With a different modelization of the human logic based on this new concept he realized that the dramatic interpretation of Arrow's impossibility theorem could have an interesting solution given by a more realistic interpretation of reality. We can see these ideas in one of his first published papers [42] titled *Arrow's theorem under fuzzy rationality*. The Arrow paradox or Arrow Theorem establishes that when voters have three or more distinct alternatives, no rank order voting system can convert the ranked preferences of individuals into a community-wide (complete and transitive) ranking while also meeting a pre-specified set of criteria. These pre-specified criteria are called unrestricted domain, non-dictatorship, Pareto efficiency, and independence of irrelevant alternatives.

From Javier Montero's point of view, the impossibilities in social choice given by the Arrow theorem are based on the fact that he used Aristotelian logic in the decision making aggregation process. In addition to the importance of the analysis of the decision making process, here also appear other ideas that will follow Javier's life until now as the ideas of *rationality*, *consistency* or *fuzzy opinion* among many others.

From then until now, group decision making as an aggregation process has been one of the main topics in Javier's research ([43, 46, 47, 49, 52, 36]) and his ideas have been the seed of many works in which the fuzzy sets has been used as a fundamental tool in group decision making and his aggregation processes (see for example [56, 54, 53, 57, 58, 55] among many others).

Reading time

Javier looks his clock after finishing the lecture. It's 6 p.m. and now the university starts to get empty. Everything is more *quiet and it is a good time to read something interesting, to navigate on the internet looking for new ideas and to think about them. In this period of the day, Javier structures his ideas about research and something more. . .*

Probably, one of the main criticism of Javier Montero about probability theory is the statistical independence. Independence is usually assumed between variables, objects, decision makers, criteria in many statistical inference problems, data analysis, machine learning, soft computing among others disciplines. Javier looks the reality and always says the same, the reality is too complex to make these strong assumptions. It is clear that sometimes, you need to make assumptions in order to be operational and efficient, but you never have to forget that the objects, variables, criteria decision makers are always related in some way. During many years, Javier has put many effort to relax these conditions and assumptions trying to be operational but also realistic with his reality view. The variables, objects, items, criteria or preferences have a structure and can not be considered as independent units of information. Now I will try to summarize some of his works in which he tried to break the walls of these assumptions.

1. **Structures in Fuzzy Sets.** The meta fuzzy sets concept defined in [51] by Javier, Humberto and myself is an example of this situation. In this paper, the authors propose an interesting solution to the existing controversy between intuitionistic fuzzy sets and interval valued fuzzy sets. From a mathematical point of view both models are equivalent. Nevertheless, both models are very different when viewed as the result of a classification problem. In that paper, it is assumed that fuzzy sets are defined from a well-defined universe of objects into a valuation space where a particular structure (a graph) is being defined, in such a way that each element of the considered universe has a degree of membership with respect to each state in the valuation space. After taking into account the structure of these fuzzy sets, a natural explanation of the different visions underlying Atanassov's model and interval valued fuzzy sets is given by the authors.

2. **Structures in Preferences.** From his first papers [16, 17] until the last ones (see for example [33, 34, 19, 20, 21]) the structure and the representation of the preferences is a key issue to understand and explain the decision making and classification problems. Just to put an example of one of his last papers published in this topic, in [21], the representation of the preferences is based on the neutrality concept in between opposite poles, such that a basic type of structure is used to characterize in logical terms the concepts and the knowledge that they generate. They modeled the meaning of concepts by paired structures, and apply these structures for learning and building the different meanings of preference for decision making.

3. **Structures in Classification problems: classes**. The classes of a classification problem always present a structure that permits to relate them. It is necessary to incorporate this inherent structure in order to build a good classification system (see for example [3, 4] or [59, 60, 61, 31]). This fact will allow to improve the learning process that permits to build the membership functions of each class and also the class itself.

4. **Structures in Image Analysis.** In addition to the classes, the units of information that have to be classified present some relations and structure among them. Pixels (in case this is the unit of information) present an inherent structure that has to be taken into account in the classification process. Only considering the spectral information of each pixel in an independent way is not always a good approach ([25, 26]).

5. **Structures in Aggregation process.** Aggregation processes have to take into account the meaning of the information that is aggregated. This fact can be observed in the aggregation process that appears in classification systems (see [3, 4, 5]). In addition to this, the properties that usually are defined for aggregation operators should be defined according to the structure of the information that is aggregated. In this sense, in [64, 65, 30], Javier presented the concept of consistency and stability when the information units that have to be aggregated present an hierarchical structure, or linear structure. Also the idea of recursivity or associativity of a global aggregation operator has to take into account the structure of the problem ([5]).

Going Home: Final conclusions

Javier looks through his window and it is dark outside. The sun is falling down in the mountains and now it is time to go home and think about the things that he will have to do tomorrow. After a hard day of work it is time to relax and enjoy with his family the rest of the day.

To conclude this work, I would like to say that Javier's human qualities and personality make him a extraordinary man. Probably this is one of the reasons why many people want to work with him, even with his limited time. I would also like to thank the EUSFLAT magazine editors for giving me the opportunity to write this paper. The content of this paper is, of course, mine.

Gomez and Montero in San Francisco in 2005.

References

[1] A. Amo, J. Montero, E. Molina, Representation of consistent recursive rules. European Journal of Operational Research 130 (2001), 29-53.

[2] A. Amo, D. Gómez, J. Montero, G. Biging, Relevance and redundancy in fuzzy classification systems. Mathware and Soft Computing 8 (2001) 203-216.

[3] A. del Amo, J. Montero and G. Biging, Classifying pixels by means of fuzzy relations. Int. J. General Systems 29 (2000) 605-621.

[4] A. Amo, J. Montero, A. Fernández, M. López, J. Tordesillas and G. Biging, Spectral fuzzy classification: an application. IEEE Trans.Syst. Man and Cyb. C 32 (2002) 42-48.

[5] A. Amo, J. Montero, G. Biging and V. Cutello, Fuzzy classification systems. European Journal of Operational Research 156 (2004) 459-507.

[6] J.K. Arrow, Social choice and individual values. Wiley, New York (1951)

[7] H. Bustince, F. Herrera and J. Montero, Fuzzy Sets and Their Extensions: Representation, Aggregation and Models. Intelligent Systems from Decision Making to Data Mining, Web Intelligence and Computer Vision (2008).

[8] H. Bustince, J. Montero, R. Mesiar, Migrativity of aggregation functions. Fuzzy Sets and Systems 160 (2009) 766-777.

[9] H. Bustince, T. Calvo, B. de Baets, J. Fodor, R. Mesiar, J. Montero, D. Paternain and A. Pradera, A class of aggregation functions encompassing two-dimansional OWA operators. Information Sciences 180 (2010) 1977-1989.

[10] H. Bustince, J. Fernandez, R. Mesiar, J. Montero and R. Orduna, Overlap functions. Nonlinear Analysis-Theory Methods and Applications 72 (2010) 1488-1499.

[11] H. Bustince, M. Pagola, R. Mesiar, E. Hullermeier, F. Herrera, J. Montero and R. Orduna, Grouping, Overlap, and Generalized Bientropic Functions for Fuzzy Modeling of Pairwise Comparisons. IEEE Transaction on Fuzzy Systems 20 (2012) 405-415.

[12] V. Cutello and J. Montero, A characterization of rational amalgamation operations. IJAR 8 (1993) 325-344.

[13] V. Cutello, J. Montero, Recursive families of OWA operators. Proceedings FUZZ-IEEE Conference - IEEE Press, Piscataway (1994), 1137-1141.

[14] V. Cutello, J. Montero, Hierarchical aggregation of OWA operators: basic measures and related computational problems. Int. J. Uncertainty Fuzzinesss Knowl. based Syst. 3 (1995), 17-26.

[15] V. Cutello and J. Montero, Equivalence and compositions of fuzzy rationality measures. Fuzzy Sets and Systems 85 (1997) 31-43.

[16] V. Cutello and J. Montero, Aggregation operators for fuzzy rationality measures. Studies in Fuzziness and Soft Comp. 12 (1998) 98-105.

[17] V. Cutello and J. Montero, Non deterministic aggregation operators and systems. Int. J. Intelligent Systems 13 (1998) 181-192.

[18] V. Cutello, J. Montero, Recursive connective rules. International Journal of Intelligent Systems 14 (1999) 3-20.

[19] C. Franco, J.T. Rodríguez and J. Montero, An ordinal approach to computing with words and the preference-aversion model. Information Sciences. 258 (2014) 239-248.

[20] C. Franco, J. Montero and J.T. Rodríguez, Relevance in preference structures. Advances in Intelligent Systems and Computing 214 (2014) 117-125.

[21] C. Franco, J.T. Rodríguez and J Montero, Building the meaning of preference from logical paired structures. Knowledge-Based Systems 83 (2015) 32-41.

[22] L. Garmendia, A. Salvador and J. Montero, Computing a T-transitive low approximation or opening of a proximity relation. Fuzzy Sets and Systems 160 (2009) 2097-2105.

[23] D. Gómez, J. Montero, A discussion of aggregation functions. Kybernetika 40 (2004) 107-120.

[24] D. Gómez, J. Montero, J. Yáñez, J. G. Pachón and V. Cutello, Crisp dimension theory and valued preference relations. International Journal of General Systems 33 (2004) 115-131.

[25] D. Gómez, J. Montero and J. Yáñez, A coloring algorithm for image classification. Information Sciences 176 (2006) 3645-3657.

[26] D. Gómez, J. Montero, J. Yáñez and C. Poidomani, A graph coloring approach for image segmentation. OMEGA International Journal of Management Science 35 (2007) 173-183.

[27] D. Gómez and J. Montero, Fuzzy sets in remote sensing classification. Soft Comptuing 12 (2008) 243-249.

[28] D. Gómez, J. Montero and G. Biging, Improvements to remote sensing using fuzzy classification, graphs and accuracy statistics. Pure and Applied Geophysics 165 (2008) 1555-1575.

[29] D. Gómez, G. Biging and J. Montero, Accuracy statistics for judging soft classification. International Journal of Remote Sensing 29 (2008) 693-709.

[30] D. Gómez, K. Rojas, J.T. Rodríguez, J. Montero, G. Beliakov, Consistency and stability in aggregation operators: An application to missing data problems. International Journal of Computational Intelligence Systems 7 (2014) 595-604.

[31] D. Gómez, E. Zarrazola, J. Yáñez and J. Montero, A Divide-and-Link algorithm for hierarchical clustering in networks. Information Sciences 316 (2015) 308-328.

[32] D. Gómez, J.T. Rodríguez, J. Montero, H. Bustince, E. Barrenechea, N dimensional overlap functions. Fuzzy sets and Sytems. In press.

[33] J. González-Pachón, D. Gómez, J. Montero and J. Yáñez, Soft dimension theory. Fuzzy Sets and Systems 137 (2003) 137-149.

[34] J. González-Pachón, D. Gómez, J. Montero and J. Yáñez, Searching for the dimension of binary valued preference relations. International Journal Approximate Reasoning, 33 (2003) 137-157.

[35] L.W. Fung adn K.S. Fu, An axiomatic approach to rational decision making in a fuzzy environment. In: Zadeh LA et al (eds) Fuzzy sets and their applications to cognitive and decision processes. Academic, New York (1975) pp 227-256.

[36] L. Martínez and J. Montero, Challenges for improving consensus reaching process in collective decisions. New Maths and Natural Computation 3 (2007) 203-217.

[37] J. Montero, A note on Fung Fu's theorem. Fuzzy Sets and Systems 17(1985) 259 269.

[38] J. Montero and J. Tejada, Some problems on the definition of fuzzy preference relation. Fuzzy Sets and Systems 20 (1986), 45-53.

[39] J. Montero, Measuring the rationality of a fuzzy preference relation. BUSEFAL 26 (1986) 75-82.

[40] J. Montero and J. Tejada, Fuzzy preferences in decision making. Lecture Notes in Computer Science 286 (1987) 144-150.

[41] J. Montero, Social welfare functions in a fuzzy environment. Kybernetes 16 (1987) 241-245.

[42] J. Montero, Arrow's theorem under fuzzy rationality. Behavioral Science 32 (1987), 267-273.

[43] J. Montero, Aggregation of fuzzy opinions in a fuzzy group. Fuzzy Sets and Systems 25 (1988) 15 20.

[44] J. Montero and J. Tejada, A necessary and sufficient condition for the existence of Orlovsky's choice set. Fuzzy Sets and Systems 26 (1988) 121-125.

[45] J. Montero, Fuzzy coherent systems. Kybermetes 17 (1988) 28-33.

[46] J. Montero, Single-Peakedness in Weighted Aggregation of Fuzzy Opinions in a Fuzzy Group. Multiperson Decision Making Models Using Fuzzy Sets and Possibility Theory Theory and Decision Library 18 (1990) 163-171.

[47] J. Montero, V. Cutello, A model for amalgamation in group decision making. NASA CONF. PUB. 10112, vol. 1. (1992) 215-223.

[48] J. Montero, Rational aggregation rules. Fuzzy Sets and Systems 62 (1994) 267-276.

[49] J. Montero, Aggregation of fuzzy opinions in a nonhomogeneous group. Fuzzy Sets and Systems 25 (1997) 15-20.

[50] J. Montero, J. Tejada and V. Cutello, A general model for deriving preference structures from data. European Journal of Operational Research. 98 (1997) 98-110.

[51] J. Montero, D. Gómez, H. Bustince, On the relevance of some families of fuzzy sets. Fuzzy Sets and Systems 158 (2007) 2429-2442.

[52] J. Montero, V. López and D. Gómez, The role of fuzziness in decision making. Studies in Fuzziness and Soft Comp. 215 (2007) 337-349.

[53] F. Herrera, E. Herrera-Viedma, J.L. Verdegay, Choice processes for non-homogeneous group decision making in linguistic setting. Fuzzy Sets and Systems 94 (1998) 287-308.

[54] D. Dubois, J.L. Koning, Social choice axioms for fuzzy set aggregation. Fuzzy Sets and Systems 43 (1991) 257-274.

[55] F.J. Cabrerizo, I.J. Pérez and E. Herrera-Viedma, Cover image Managing the consensus in group decision making in an unbalanced fuzzy linguistic context with incomplete information. Knowledge-Based Systems 23 (2010) 169-181.

[56] J. L. Marichal, Aggregation operators for multicriteria decision aid, (1988) Phd. Thesis.

[57] F. Chiclana, E. Herrera-Viedma, F. Herrera and S. Alonso, Some induced ordered weighted averaging operators and their use for solving group decision-making problems based on fuzzy preference relations. European Journal of Operational Research 182 (2007) 383-399.

[58] R.J. Li, Fuzzy method in group decision making. Computers and Mathematics with Applications 38 (1999) 91-101.

[59] J.T. Rodríguez, B. Vitoriano and J.Montero, A general methodology for inductive fuzzy-rules building and its application to natural disaster management. Computers and Operations Research 39 (2012) 853-873.

[60] J.T. Rodríguez, J. Montero, B. Vitoriano, Dissimilarity-based bipolar supervised classification. World Scientific Proceedings Series on Computer Engineering and Information Science 7 (2012) 894-899.

[61] J.T. Rodríguez, B. Vitoriano, D. Gómez and J. Montero, Classification of Disasters and Emergencies under Bipolar Knowledge Representation. Atlantis Computational Intelligence Systems Series 7 (2013) 209-232.

[62] J.T. Rodríguez, E. Turunen, D. Ruan, J. Montero, Another paraconsistent algebraic semantics for Lukasiewicz-Pavelka logic. Fuzzy sets and Systems 242 (2014) 132-147.

[63] K. Rojas, D. Gómez, J.T. Rodríguez, J. Montero, Some properties of consistency in the families of aggregation functions. Advances in Intelligent and Soft Computing 107:169-176 (P. Melo-Pinto, P. Couto, C. Serodio, J. Fodor, B. De Baets, Eds.) Springer, Berlin (2011).

[64] K. Rojas, D. Gómez, J.T. Rodríguez, J. Montero, Strict stability in aggregation operators, Fuzzy sets and Systems 228 (2013) 44-63.

[65] K. Rojas, D. Gómez, J.T. Rodríguez, J. Montero. A. Valdivia, F. Paiva, Development of child's home environment indexes based on consistent families of aggregation operators with prioritized hierarchical information. Fuzzy sets and Systems 241 (2014) 41-60.

[66] J. Yáñez and J. Montero, A poset dimension algorithm. Journal of algorithms 30 (1999) 185-208.

[67] G. Zhang, J. Lu, J. Montero and Y. Zeng, Model, solution concept, and Kth best algorithm for linear tri-level programming. Information Sciences 180 (2010) 799-802.

SCIENTIFIC REPORT

Soft Computing and Computer Vision in Forensic Identification

B.R. Campomanes-Álvarez[1], C. Campomanes-Álvarez[2], E. Bermejo[2], A. Valsecchi[1], O. Ibáñez[2], S. Damas[1] and O. Cordón[1,2]
[1]European Centre for SoftComputing, Mieres, Spain
{rosario.campomanes, andrea.valsecchi, sergio.damas, oscar.cordon}@softcomputing.es
CITIC and Dept. of Computer Science and Artificial Intelligence, University of Granada,Spain
{oscar.ibanez, enrique.bermejo, ocordon}@decsai.ugr.es

FORENSIC IDENTIFICATION

Human identification is of paramount importance in our society. It does not only resolves serious legal and social predicaments, but it also provides a resolution to grieving families who need closure to their sadness. During the last two decades techniques like DNA or fingerprints have been employed in many identification scenarios. However, the application of these methods fail when there is not enough information available, ante-mortem (AM) or post-mortem (PM), due to the lack of data (second DNA sample), or due to the state of preservation of the corpse. While the skeleton usually survives both natural and non-natural decomposition processes (fire, salt, water, etc.), the soft tissue progressively degrades and is lost, as is the most frequent case in disaster victim identification scenarios. In fact, the experience of several practitioners in the latter scenarios indicates the poorer effectiveness of DNA analysis (around 3% of the identifications) and dactyloscopy (15-25%) against skeleton-based identification techniques (70-80%) [1].

Skeleton-based identification methods employed by forensic anthropologists, odontologists, and pathologists are crucial in AM data collection and biological profiling. What is even more important, these methods represent the victim's last chance for identification using techniques such as comparative radiography or craniofacial superimposition. In this short manuscript we will review the most recent advancements made by the University of Granada and the European Centre for Soft Computing on the automation of the Craniofacial Superimposition technique [2]. Based on soft computing and computer vision techniques, we have developed the first computer-aided automatic system in the field of skeleton-based identification, Face2Skull software [27]. These research has been supported by three national and two regional research projects, as well as by an European project, and has resulted in an international filled patent[1].

CRANIOFACIAL SUPERIMPOSITION

Craniofacial superimposition (CFS) [2, 3, 4, 5, 6, 7] is one of the most relevant skeleton-based identification techniques. It involves the process of overlaying a skull image (or a skull 3D model) with a number of AM images of an individual and the analysis of their morphological correspondence. By projecting photographs of the skull and of the missing person on top of each other, the practitioner can try to establish whether they correspond to the same individual. Three consecutive stages for the whole CFS process have been distinguised in [8]:

- Acquisition and processing of the face and the skull photographs/models. In some approaches, this step also involves the location of anatomical landmarks on the skull and the face.

- Skull-face overlay (SFO), which focuses on achieving the best possible superimposition of either an image, video-frame or a 3D model of a physical skull, and a single AM image of a missing person. This process is iteratively repeated for each available candidate photograph, obtaining different overlays. Skull-face overlay thus refers to what traditionally has been known as the adjustment of the skull size and its orientation with respect to the facial photograph [2].

- Skull-face overlay assessment and decision making, in which the degree of support that the skull and the available photograph belong to the same person or not (exclusion) is determined. This decision is guided by different criteria studying the relationship between the skull and the face: the morphological correlation, the matching between the corresponding landmarks according to the soft tissue depth, and the consistency between asymmetries.

An important historical limitation of the CFS technique is the absence of a common methodology accepted worldwide. Experts try to solve the CFS problem by applying a specific approach considering their knowledge and the available technologies. During the SFO stage, most practitioners follow a trial-and-error method until they attain a good enough superimposition. The adjustment of the skull size and its orientation with respect to the facial photograph is a very challenging and time-consuming part of CFS. That task can take

[1]Cordón, O., Damas, S., Ibáñez, O., Santamaría, J., Alemán, I., Botella, M. Sistema de Identificación Forense por Superposición Craniofacial Basado en Soft Computing (Forensic Identification System Using Craniofacial Superimposition Based on Soft Computing). Publication No.: WO/2011/01274. Publication date: 03/02/2011. International application No.: PCT/ES2010/00350. International filling date: 30/07/2010. Priority Data: P200901732 30.07.2009 ES. Designated States: International. Owning Institutions: Foundation for the Advancement of Soft Computing, University of Granada. Web: `http://www.wipo.int/patentscope/search/en/WO2011012747`.

hours to arrive at the best possible fit [9, 10]. Hence, a systematic and automatic method for CFS is a real need in forensic anthropology [10]. Fortunately, the European project MEPROCS[2], participated by 25 labs from 17 countries, has recently concluded with the development of the first standard in the field, including good and bad practices, sources of error and uncertainties, technological requirements and desirable features, and finally a common scale for the craniofacial matching evaluation [11].

SOFT COMPUTING AND COMPUTER VISION FOR CRANOFACIAL SUPERIMPOSITION

Computational methods as computer vision (CV) and soft computing (SC) can be extremely useful for automating the CFS process. Computer vision includes techniques for processing, analyzing, segmenting, and registering image data in an automatic way [12]. Within CV, image registration (IR) aims to find a geometric transformation that overlays two images taken under different conditions (at different times, from different viewpoints, and/or by different sensors) [13]. Meanwhile, SC aims to design intelligent systems able to process uncertain, imprecise, and incomplete information [14]. Soft computing methods applied to real-world problems often provide more robust and tractable solutions than those obtained by more conventional mathematical techniques. Two of the main SC techniques are fuzzy logic and fuzzy set theory, which extend classical logic to provide a conceptual framework for knowledge representation under imprecision and the consequent uncertainty [15], and evolutionary algorithms (EAs), which are powerful bio-inspired search and optimization tools to automate problem solving in areas such as modeling, simulation, or global optimization [16, 17].

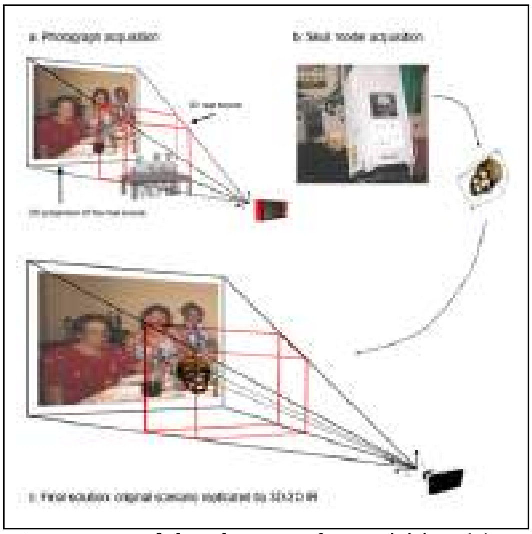


Figure 1. Moment of the photograph acquisition (a) and skull 3D modeling (b). Orientation of the skull 3D model in the photograph by a location, scaling and perspective projection (c).

Computer vision has been employed to automate the time-cosuming and prone-error SFO stage. The most natual way to do so is to replicate the original scene in which the photograph was taken. From the CV point of view, this involves a 3D-2D IR problem [13]. Figure 1 illustrates the different moments and elements that have to be considered to model the SFO problem following an IR approach. Figure 1a shows the moment of the photograph acquisition with the real scene enclosed by the 3D polygon in red. The skull 3D model acquisition by using a range scanner as depicted in Fig. 1b, and the replication of the original scenario by means of a 3D-2D IR task is shown in Fig. 1c.

The aforementioned IR scenario shows a high complexity. It has the problem of presenting incomplete and vague information guiding the process (matching of two different objects, a skull and a face), and requires searching in a huge solution space with many local minima to identify the registration transformation parameters. Therefore, SC techniques, advanced EAs in particular, have been designed to cope with this really complex optimization task [18, 19, 20].

SC has been also an essential set of tools to model SFO sources of uncertainty. There is an inherent uncertainty that is associated with the matching of two different kinds of objects that are involved in the process, i.e., a skull and a face. In addition, there is also an uncertainty that is associated with the 3D-2D overlay process that tries to superimpose a 3D model over a 2D image. This way, we have analyzed and modeled the landmark location and matching uncertainties using fuzzy sets and fuzzy distances significantly outperforming previous results [21, 22]. The landmark location uncertainty is related to the extremely difficult task to locate the points in an invariable place since the definition of any anthropometric landmark is imprecise in its own [23]. It also refers to the difficulty to locate landmarks with the accuracy required for the automatic overlay of a 3D skull model and a 2D face photo. The ambiguity may arise from reasons such as variation in shade distribution depending on light condition during photographing, unsuitable camera focusing, poor image quality, face pose in the photograph, partial or whole landmark occlusion, etc [6]. Finally, the matching uncertainty refers to to the imprecision that is involved in the matching of two sets of landmarks corresponding to two different objects: a face and a skull. There is a clear partial matching situation. The correspondence between facial and cranial anthropometric landmarks is not always symmetrical and perpendicular; some landmarks are located in a higher position in the alive person face than in the skull, and some others do not have a directly related landmark in the other set. Besides, the facial soft tissue depth varies for each cephalometric landmark, as well as for different person groups (based on age, race, and gender) [7, 24].

ILLUSTRATIVE RESULTS

Some experiments have been accomplished to illustrate the behavior of the automatic SFO method based on CV and SC proposed in [21, 25, 22]. Every superimposition reported in the study has been directly obtained by our automatic method always taking less than 4 minutes. The accuracy of our method has been tested using a ground truth data set [26] showing an outstanding performance with an average error of around 5mm [22]. However, in some cases (in the worst case the error reached 12 mm) the manual refinement

[2]http://www.meprocs.eu/

of the resulting SFO by the expert using the software toold could be required.

Figure 2 shows the superimpositions obtained for two photographs of two different cases provided by the Physical Anthropology Lab of the University of Granada. The outcomes achived by the automatic SFO method are reported in phantom mode. In particular, points in red color (in gray in the black and white version) correspond to the cranial landmarks after overlaying the skull 3D model on the photograph. Points colored in green (light gray in the black and white version) are the facial landmarks marked by the expert in the photograph.

Notice that, each pair of corresponding landmarks is expected to have a partial matching according to the soft tissue depth and the effect of the projection on a 2D image. Nevertheless, the evaluation of the degree of matching between the skull and the face corresponds to the forensic expert within the decision making stage.

Figure 2. Automatically obtained superimpositions for female (left) and male (right) case in phantom mode.

CONCLUSIONS AND FUTURE WORKS

Althoug applied for more than one hundred years, CFS is still a very challenging identification technique from a technological perspective. Our novel automatic proposal [21, 25, 22], based on CV and SC, performs the second CFS stage (SFO) in the most natural way by replicating the original scene in which the photograph was taken. Analyzing the achieving overlays, it obtains good superimpositions in a complete automatic, reproducible and objective approach. The next step is the modelization and automation of the last stage, the decision making, by means of a fuzzy decision support system. In the third stage, forensic experts evaluate spatial and morphological skull-face relationships focusing on certain regions that demonstrated to be more discriminative. The final decision is provided in terms of strong, moderate or limited support to the assertion that the skull and the facial image belong to the same person or not [11]. This is a subjective process that can depend on the forensic expert's skills and the quantity and quality of the used materials. Hence, having a decision support system that can assist forensic anthropologists to take the final decision in objective way can be interesting. Our long-term, very complex goal is to design such a decision support system based on the evaluation of the said spatial and morphological relations. This system will provide a numeric index as output, aiming to support to forensic anthropology to take the CFS final decision.

Acknowledgements

This work has been supported by the following institutions: the Principality of Asturias Government under the project with reference CT13-55, the Spanish MINECO under the SOCOVIFI2 project (refs. TIN2012-38525-C02-01/02, `http://www.softcomputing.es/socovifi/`), and the Andalusian Department of Innovación, Ciencia y Empresa under project TIC2011-7745. Mrs. C. Campomanes Alvarez's work has been supported by Spanish MECD FPU grant AP-2012-4285. Dr. Ibáñez's work has been supported by Spanish MINECO Juan de la Cierva Fellowship JCI-2012-15359.

References

[1] Beauthier JP, De Valck E, Lefevre P, De Winne J (2009) Mass Disaster Victim Identification: The Tsunami Experience. The Open Forensic Sci J 2:54-62.

[2] Yoshino M (2012) Craniofacial superimposition. In: Wilkinson C, Rynn C (eds) Craniofacial Identification, University Press, Cambridge, pp 238-253.

[3] Iscan MY (1981) Integral forensic anthropology. Practicing Anthropology, 3:21-30.

[4] Krogman WM, Iscan MY (1986) The human skeleton in forensic medicine. 2nd ed Charles C. Thomas, Springfield.

[5] Iscan MY (1993) Introduction to techniques for photographic comparison. In: Iscan MY and Helmer R (eds) Forensic Analysis of the Skull: Craniofacial Analysis, Reconstruction, and Identification, Wiley, New York, pp. 57-90.

[6] Taylor J, Brown K (1998) Superimposition Techniques. In: Clement J and Ranson D (eds) Craniofacial Identification in Forensic Medicine, Arnold, London, pp. 151-164.

[7] Stephan CN (2009) Craniofacial identification: Techniques of facial approximation and craniofacial superimposition. In: Blau S, Ubelaker DH (eds) Handbook of Forensic Anthropology and Archaeology, Left Coast Press, Walnut Creek, 25:304-321.

[8] Damas S, Cordón O, Ibáñez O, Santamaría J, Alemán I, Botella M et al (2011) Forensic identification by computer-aided craniofacial superimposition: A survey. ACM Computing Surveys 43(4):27.

[9] Fenton TW, Heard AN, Sauer J (2008) Skull-photo superimposition and border deaths: identification through exclusion and the failure to exclude. J Forensic Sci 53:34-40.

[10] Ubelaker DH (2000) A history of Smithsonian-FBO collaboration in forensic anthropology, especially in regard to facial imagery [Abstract]. Forensic Sci Commun, 2.

[11] Damas S, Wilkinson C, Kahana T, Veselovskaya E, Abramov A, Jankauskas R, Jayaprakash P, Ruiz E,

Navarro F, Huete M, Cunha E, Cavalli F, Clement J, Leston P, Molinero F, Briers T, Viegas F, Imaizumi K, Humpire D, Ibáñez O (2015) Study on the performance of different craniofacial superimposition approaches (ii): best practices proposal, Forensic Sci Int. In press.

[12] Sonka M, Hlavac V, Boyle R (2007) Image Processing, Analysis, and Machine Vision 3rd ed. Thomson-Engineering, Toronto.

[13] Zitovà B, Flusser J (2003) Image registration methods: a survey. Image Vis Comput, 21:977-1000.

[14] Bonissone PP (1997) Soft computing: the convergence of emerging reasoning technologies. Soft Comput 1(1):6-18.

[15] Zadeh LA (1994) Soft computing and fuzzy logic. IEEE Softw, 11:48-56.

[16] Bäck T, Fogel DB, Michalewicz Z (eds) (1997) Handbook of Evolutionary Computation 1st ed. IOP Publishing Ltd., Bristol.

[17] Eiben AE, Smith JE (2003) Introduction to Evolutionary Computation. Natural Computing Series. Springer, Heidelberg.

[18] Ibáñez O, Cordón O, Damas S, Santamaría J (2009) An experimental study on the applicability of evolutionary algorithms to craniofacial superimposition in forensic identification. Inf Sci, 79:3998-4028.

[19] Ibáñez O, Cordón O, Damas S, Santamaría J (2012) An advanced scatter search design for skull-face overlay in craniofacial superimposition. Expert Syst Appl, 39(1):1459-1473.

[20] Ibáñez O, Cordón O, Damas S (2012) A cooperative coevolutionary approach dealing with the skull-face overlay uncertainty in forensic identification by craniofacial superimposition. Soft Comp, 18:797-808.

[21] Ibáñez O, Cordón O, Damas S, Santamaría J (2011) Modeling the skull-face overlay uncertainty using fuzzy sets, IEEE Trans Fuzzy Syst, 16:946-959.

[22] Campomanes-Álvarez BR, Ibáñez O, Campomanes-álvarez C, Damas S, Cordón O (2015) Modeling facial soft tissue thickness for automatic skull-face overlay. IEEE Trans Inf Forensics Security. In press.

[23] Campomanes-álvarez BR, Ibáñez O, Navarro F, Alemán I, Cordón O, Damas S (2015) Dispersion assessment in the location of facial landmarks on photographs. Int J Legal Med, 129(1):227-236.

[24] Gordon GM, Steyn M (2012) An investigation into the accuracy and reliability of skull-photo superimposition in a South African sample. Forensic Sci Int 216(1-3):198.e1-6.

[25] Campomanes-Álvarez BR, Ibáñez O, Navarro F, Botella M, Damas S, Cordón O (2015) Computer Vision and Soft Computing for Automatic Skull-Face Overlay in Craniofacial Superimposition. Forensic Sci Int, 245:77-86.

[26] Ibáñez O, Cavalli F, Campomanes-Álvarez BR, Campomanes-Álvarez C, Valsecchi A, Huete MI (2015) Ground truth data generation for skull-face overlay. Int J Legal Med, 129(3):569-81.

[27] Face2SkullTM software. `http://www.face2skull.com/` [Online accessed 31-may-2015]

SCIENTIFIC REPORT

V-ANFIS for Dealing with Visual Uncertainty for Force Estimation in Robotic Surgery

Angelica I. Aviles
Universitat Politècnica de Catalunya - BarcelonaTech

Accurate and robust estimation of applied forces in Robotic-Assisted Minimally Invasive Surgery is a very challenging task. Many vision-based solutions attempt to estimate the force by measuring the surface deformation after contacting the surgical tool. However, visual uncertainty, due to tool occlusion, is a major concern and can highly affect the results' precision. In this paper, a novel design of an adaptive neuro-fuzzy inference strategy with a voting step (V-ANFIS) is used to accommodate with this loss of information. Experimental results show a significant accuracy improvement from 50% to 77% with respect to other proposals.

SCIENTIFIC REPORT

A fuzzy edge-based image segmentation approach

Carely Guada
Complutense University of Madrid

Segmentation is a technique used to partition the pixels of an image in regions, where each detected region or object is delimited by their boundaries. However, sometimes these boundaries are not sharp and clear. In the IFSA/EUSFLAT World Congress 2015's Special Session of Image Processing with Fuzzy Techniques, I will present an approach to define the concept of Fuzzy Image Segmentation, which has not been previously studied in the literature. This problem faces the blurred nature of objects in a digital image, as well as of the borders between them. The concept of fuzzy image segmentation proposed here is based on the fuzzy boundary class over the edge set of the image network. The concept of crisp image segmentation into a fuzzy framework is extended in this work. Based on this new approach, a specific visualization of a fuzzy segmented image is proposed. It is important to distinguish that even when using fuzzy-based techniques, the resulting segmentation uses to be crisp. Some computational experiences which give a fuzzy segmented image are presented. The results have been obtained firstly by segmenting an image in a series of hierarchical levels, and subsequently developing an aggregation process that leads to obtain a blurred graduation of the segmented image. In these experiences, graduations in the lines used to delimit the fuzzy segmented objects become more similar to a human segmentation. Also, in this way we deal with the noise or imperfections that an image could have.

SCIENTIFIC REPORT

Fuzzy Clustering and Prediction of Electricity Demand Based on Household Characteristics

Joaquim Viegas
Universidade de Lisboa

The electricity market has been significantly changing in the last decade. The deployment of smart meters is enabling the logging of huge amounts of data relating to the operations of utilities with the potential of being translated into valuable knowledge on the behaviour of consumers. This work proposes a methodology for predicting the typical daily load profile of electricity usage based on static data using fuzzy clustering and modelling. The methodology intends to:

(1) determine consumer segments based on the metering data using the fuzzy c-means clustering algorithm, and

(2) develop Takagi-Sugeno fuzzy models in order to predict the demand profile of the consumers.

SCIENTIFIC REPORT

Construction of admissible orders for intervals

L. De Miguel[1], H. Bustince[1], J. Fernandez[1], A. Kolesárová[2], R. Mesiar[2] and B. De Baets[3]
[1]Universidad Publica de Navarra, Pamplona, Spain, e-mail: laura.demiguel@unavarra.es
[2]Slovak University of Technology, Bratislava, Slovakia
[3]Ghent University, Ghent, Belgium

Linear order are required in some theoretical and applied developments. Examples of such are Choquet and Sugeno integrals (in a theoretical context), or decision making and fuzzy classification (in an applied one).

However, linear ordering becomes significantly more complicated when it applies to non-scalar or intervalar data. In particular, we study linear orders for m-tuples of closed subintervals.

We only consider linear orders ($\preceq$) which refine the partial order on $\mathbb{R}^{2m}$ i.e., linear orders which satisfy that given $([\underline{x}_1,\overline{x}_1],\ldots,[\underline{x}_m,\overline{x}_m]),([\underline{y}_1,\overline{y}_1],\ldots,[\underline{y}_m,\overline{y}_m])$ such that $\underline{x}_i \leq \underline{y}_i$ and $\overline{x}_i \leq \overline{y}_i$ for all $i \in \{1,\ldots,m\}$ then $([\underline{x}_1,\overline{x}_1],\ldots,[\underline{x}_m,\overline{x}_m]) \preceq ([\underline{y}_1,\overline{y}_1],\ldots,[\underline{y}_m,\overline{y}_m])$. We refer this class to admissible orders.

In the development of the work two different constructing methods are studied.

Proposition 1 *Let $\preceq_{B_1,B_2}$ be an admissible order on the set of one interval [1] and let $\sigma = (\sigma(1),\ldots,\sigma(m))$ be a permutation. The order $\preceq_{[\sigma,B_1,B_2]}$ on the set of m intervals, given by*

$$([\underline{x}_1,\overline{x}_1],\ldots,[\underline{x}_m,\overline{x}_m]) \preceq_{[\sigma,B_1,B_2]} ([\underline{y}_1,\overline{y}_1],\ldots,[\underline{y}_m,\overline{y}_m])$$

if and only if one of the following conditions holds

- $[\underline{x}_{\sigma(1)},\overline{x}_{\sigma(1)}] \prec_{B_1,B_2} [\underline{y}_{\sigma(1)},\overline{y}_{\sigma(1)}]$
- $[\underline{x}_{\sigma(1)},\overline{x}_{\sigma(1)}] = [\underline{y}_{\sigma(1)},\overline{y}_{\sigma(1)}]$ *and* $[\underline{x}_{\sigma(2)},\overline{x}_{\sigma(2)}] \prec_{B_1,B_2} [\underline{y}_{\sigma(2)},\overline{y}_{\sigma(2)}]$

...

- $[\underline{x}_{\sigma(i)}, \overline{x}_{\sigma(i)}] = [\underline{y}_{\sigma(i)}, \overline{y}_{\sigma(i)}]$ *for all* $i \in \{1, \ldots, m-2\}$ *and* $[\underline{x}_{\sigma(m-1)}, \overline{x}_{\sigma(m-1)}] \prec_{B_1,B_2} [\underline{y}_{\sigma(m-1)}, \overline{y}_{\sigma(m-1)}]$
- $[\underline{x}_{\sigma(i)}, \overline{x}_{\sigma(i)}] = [\underline{y}_{\sigma(i)}, \overline{y}_{\sigma(i)}]$ *for all* $i \in \{1, \ldots, m-1\}$ *and* $[\underline{x}_{\sigma(m)}, \overline{x}_{\sigma(m)}] \preceq_{B_1,B_2} [\underline{y}_{\sigma(m)}, \overline{y}_{\sigma(m)}]$

is an admissible order.

Definition 1 *Let* $A = (A_1, A_2, \ldots, A_{2m})$ *be* $2m$ *aggregation functions* $A_i : [0,1]^{2m} \to [0,1]$. *The* $2m$*-tuple* A *is admissible if for all m-tuples of intervals* $([\underline{x}_1, \overline{x}_1], \ldots, [\underline{x}_m, \overline{x}_m]), ([\underline{y}_1, \overline{y}_1], \ldots, [\underline{y}_m, \overline{y}_m])$,

$$A_i(\underline{x_1}, \overline{x_1}, \ldots, \underline{x_m}, \overline{x_m}) = A_i(\underline{y_1}, \overline{y_1}, \ldots, \underline{y_m}, \overline{y_m})$$

for all $i \in \{1, \ldots, 2m\}$ *if and only if*

$$([\underline{x}_1, \overline{x}_1], \ldots, [\underline{x}_m, \overline{x}_m]) = ([\underline{y}_1, \overline{y}_1], \ldots, [\underline{y}_m, \overline{y}_m]).$$

Proposition 2 *Let A be an admissible* $2m$*-tuple of aggregation functions. An admissible order* $\preceq_A$ *on the set of m intervals can be defined as*

$$([\underline{x}_1, \overline{x}_1], \ldots, [\underline{x}_m, \overline{x}_m]) \prec_A ([\underline{y}_1, \overline{y}_1], \ldots, [\underline{y}_m, \overline{y}_m])$$

if and only if there is a $k \in \{1, \ldots, m\}$ *such that*

$$A_i(\underline{x_1}, \overline{x_1}, \ldots, \underline{x_m}, \overline{x_m}) = A_i(\underline{x_1}, \overline{x_1}, \ldots, \underline{x_m}, \overline{x_m})$$

for all $i \in S = \{1, \ldots, k-1\}$ *and*

$$A_k(\underline{x_1}, \overline{x_1}, \ldots, \underline{x_m}, \overline{x_m}) < A_k(\underline{y_1}, \overline{y_1}, \ldots, \underline{y_m}, \overline{y_m}),$$

provided that $k = 1$ *induces* $S = \emptyset$.

Besides,

$$([\underline{x}_1, \overline{x}_1], \ldots, [\underline{x}_m, \overline{x}_m]) = ([\underline{y}_1, \overline{y}_1], \ldots, [\underline{y}_m, \overline{y}_m])$$

if and only if $\underline{x_i} = \underline{y_i}$ *and* $\overline{x_i} = \overline{y_i}$ *for all* $i \in \{1, \ldots, m\}$.

References

[1] H. Bustince, J. Fernandez, A. Kolesárová, R. Mesiar, Generation of linear orders for intervals by means of aggregation functions, Fuzzy Set Syst. 220(2013) 69-77.

SCIENTIFIC REPORT

Improving medical decisions under incomplete data using interval–valued fuzzy aggregation

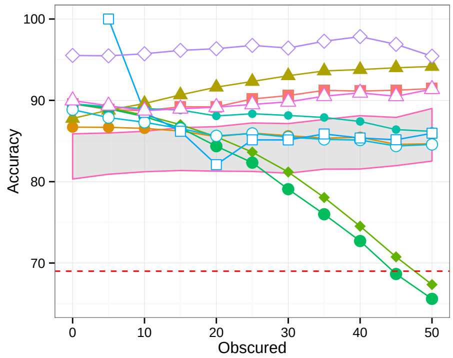
Patryk Żywica, Andrzej Wójtowicz, Anna Stachowiak, Krzysztof Dyczkowski
Faculty of Mathematics and Computer Science, Adam Mickiewicz University in Poznań Umultowska 87, 61-614 Poznań, Poland, e-mail: min@wmi.amu.edu.pl

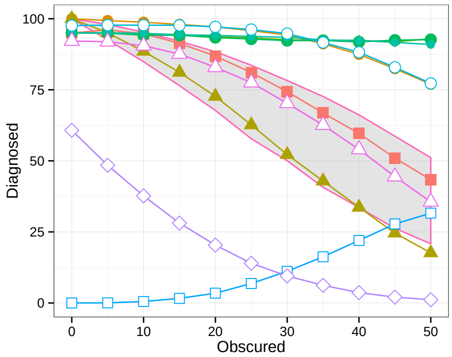
It is almost inevitable that empirical observations and real-life research produce incomplete datasets. Incomplete, or missing, data can occur for many reasons. The important point is that these reasons are natural and unavoidable, and thus the desire for complete datasets is impossible to fulfil. Clearly, missing data can have a significant effect on the conclusions that can be drawn from the data, and so it becomes a crucial issue to deal properly with missingness. In recognition of this problem, missing data analysis and decision-making under incomplete information has recently become an important area of research [1].

The present article is a contribution to the study of decision-making in the presence of incomplete information. The subject of our research is a method for supporting the medical diagnosis of ovarian tumors [2]. Since collecting all the data about a patient is often very difficult, it is crucial that the diagnostic system can give meaningful and accurate results even when some of the data is missing. We present here a novel approach that makes this possible. A key feature of our approach is that we do not use any of the known techniques for estimating missing data and data imputation, because these might significantly distort the final diagnosis. Instead, we construct a general method that makes it possible to adapt existing and well-established diagnostic methods to make them usable with incomplete data. This is achieved by interval-valued fuzzy set modelling, uncertaintification of classical methods, and finally aggregation of the incomplete results.

Our study group consisted of 268 women diagnosed and treated for ovarian tumor in the Division of Gynaecological Surgery, Poznań University of Medical Sciences, between 2005 and 2012. Among them, 62% were diagnosed with a benign tumor and 38% with a malignant tumor. The dataset is described in detail in [3].

Some results are presented in Fig. 1. Diagrams (a – accuracy) and (b – percentage of patients diagnosed) show how the aggregators and single diagnostic scales (indicated by shaded region) perform with an increasing level of missing data. The dashed horizontal line in diagram (a) indicates the accuracy of the baseline classifier.





The results presented are promising and show that the competent selection and use of aggregation methods can significantly improve the quality of decisions taken by a diagnostic system. The problem is particularly significant when the knowledge is based on incomplete information. Proper selection of the method of aggregation is essential for reducing the negative impact of data incompleteness on the quality of decisions. Because the design of an aggregation method depends on the particular problem, extensive evaluation is needed on each occasion. This can be done using our proposed method.

References

[1] R. Scerer, Multiple Fuzzy Classification Systems. Springer, 2012.

[2] k. Dyczkowski, A. Wójtowicz, et al. An intelligent system for computer-aided ovarian tumor diagnosis. In D. Filev et al., editors, Intelligent Systems 2014, volume 323 of Advances in Intelligent Systems and Computing, 335-343. Springer International Publishing, 2015.

[3] R. Moszyński, P. Żywica, et al. Menopausal status strongly influences the utility of predictive models in differential diagnosis of ovarian tumors: An external validation of selected diagnostic tools. Ginekologia Polska, 85(12): 892-899, 2014.

SCIENTIFIC REPORT

Introducing Interpolative Boolean algebra into Intuitionistic fuzzy sets

Pavle Milošević[1], Ana Poledica[1], Aleksandar Rakićević[1], Bratislav Petrović[1] and Dragan Radojević[2]
[1]University of Belgrade, Faculty of Organizational Sciences, Jove Ilića 154, Belgrade, 11000, Serbia
[2]Mihajlo Pupin Institute, Volgina 15, Belgrade, 11000, Serbia

The main idea of this paper is to introduce Interpolative Boolean algebra (IBA) as a suitable algebra for intuitionistic fuzzy sets (IFSs). IFS theory is considered as a generalization of traditional fuzzy sets. IFSs include degrees of membership, non-membership and non-determinacy and therefore offer more descriptive power comparing to conventional fuzzy logic. IBA is a $[0,1]$-realization of Boolean algebra, consistent with Boolean axioms and theorems. Boolean laws are secured by the uniquely mapping of logical functions to generalized Boolean polynomials. IBA is already utilized as a natural framework for consistent fuzzy logic in the sense of Boole.

In this paper, we present a Boolean consistent approach to IFSs by combining IFS with IBA. The concept of IFSs is fully retained, while IBA, with minor adaptations, is applied as the generalization of operations. More precisely, IFS conjunction and disjunction operations are realized using IBA generalized Boolean polynomials as follows:

$$V(x \wedge y)^{\otimes} = \left\langle a_x \otimes a_y, b_x + b_y - b_x \otimes b_y \right\rangle$$
$$V(x \vee y)^{\otimes} = \left\langle a_x + a_y - a_x \otimes a_y, b_x \otimes b_y \right\rangle$$

where generalized product ($\otimes$) can be any operator that is between Lukasiewicz and min t-norm. For the negation, we use the existing IFS operator:

$$V(\neg x)^{\otimes} = \langle b_x, 1 - b_x \rangle$$

The conventional IF calculus is obtained as the special case of our approach, when min function is used as generalized product. We also prove that the law of contradiction is thoroughly followed in IFS-IBA approach while the law of excluded middle is satisfied in three of four basic forms. The double negation rule is not preserved for the negation operator. All these facts are in accordance with the idea of intuitionism.

SCIENTIFIC REPORT

An axiomatic definition of cardinality for finite interval-valued hesitant fuzzy sets

Pelayo Quirós[1], Pedro Alonso[1], Irene Díaz[2], Vladimír Janiš[3]
[1]Department of Mathematics, University of Oviedo, Spain, e-mail: uo205956@uniovi.es
[2]Department of Computer Science, University of Oviedo, Spain
[3]Department of Mathematics, Matej Bel University, Slovak Republic

Recently, some extensions of the classical fuzzy sets are studied in depth due to the good properties that they present. Among them, in this paper

nite interval-valued hesitant fuzzy sets are the central piece of the study, as they are a generalization of more usual sets, so the results obtained can be immediately adapted to them.

In this work, the cardinality of finite interval-valued hesitant fuzzy sets is studied from an axiomatic point of view, along with several properties that this definition satisfies, being able to relate it to the classical definitions of cardinality given by Wygralak or Ralescu for fuzzy sets.

SCIENTIFIC REPORT

An analysis of the median of a fuzzy random variable based on Zadeh's extension principle

Sonia Pérez-Fernández and Beatriz Sinova
Departamento de Estadística e I.O. y D.M., Universidad de Oviedo, 33071 Oviedo, Spain

The median of a fuzzy random variable has been extended either by applying Zadeh's extension principle or by minimizing its mean distance with respect to a fuzzy number when a certain L^1 metric is considered.

This paper aims to analyze connections between both approaches along with some properties of the first one, as well as to discuss the frameworks each of the approaches better fit.

Keywords: 1-norm metric between fuzzy numbers, fuzzy random variable, median of a fuzzy number, random fuzzy number, Zadeh's extension principle.

SCIENTIFIC REPORT

On the functional equation $f(m_1(x+y)) = m_2(f(x)+f(y))$ for injective function m_2

Wanda Niemyska and Michał Baczyński
Institute of Mathematics, University of Silesia, 40-007 Katowice, ul. Bankowa 14, Poland, e-mail: wniemyska@us.edu.pl

Recently, in some considerations connected with the distributivity laws of fuzzy implications over triangular norms and conorms, the following functional equation appeared $f(\min(x+y,a)) = \min(f(x)+f(y),b)$, where a, b are finite or infinite nonnegative constants (see [1]). In [2] we have considered a generalized version of this equation in the case when both a and b are finite, namely the equation $f(m_1(x+y)) = m_2(f(x)+f(y))$, where m_1, m_2 are functions defined on some finite intervals of $\mathbb{R}$ satisfying additional assumptions. In this article we consider the above equation when m_1, m_2 are defined on some finite or infinite sets and satisfy only one additional assumption: m_2 is injective.

References

[1] M. Baczyński and B. Jayaram, *On the distributivity of fuzzy implications over nilpotent or strict triangular conorms*, IEEE Trans. Fuzzy Syst. **vol. 17, no. 3** (2009), 590–603.

[2] M. Baczyński, W. Niemyska and T. Szostok, *On a functional equation related to the distributivity of fuzzy implications over triangular norms and conorms*, Kybernetika **vol. 50, no. 4** (2014), 679–695.

[3] M. Kuczma, *An Introduction to the Theory of Functional Equations and Inequalities. Cauchy's Equation and Jensen's Inequality*, Polish Scientific Publishers, 1985.

SCIENTIFIC REPORT

On the Recall Capability of Recurrent Exponential Fuzzy Associative Memories Based on Similarity Measures

Marcos Eduardo Valle and Aline Cristina de Souza
University of Campinas, Department of Applied Mathematics,
CEP 13083-859. Campinas – SP, Brazil
E-mails: valle@ime.unicamp.br, s.alinedesouza@gmail.com

Abstract. Recurrent exponential fuzzy associative memories (RE-FAMs) are non-distributive memory models derived from the multivalued exponential recurrent associative memory (MERAM) of Chiueh and Tsai. A RE-FAM defines recursively a sequence of fuzzy sets obtained by a weighted average of the fundamental memories. In this paper, we show that the output of a single-step RE-FAM can be made as close as desired to a certain convex combination of the fundamental memories most similar to the input. This paper also addresses the storage and recall capability of RE-FAMs. Precisely, computational experiments reveal that RE-FAMs can be effectively used for the retrieval of gray-scale images corrupted by either Gaussian noise or salt and pepper noise.

Keywords. Associative memory, recurrent neural network, fuzzy system, gray-scale image processing.

1. Introduction

Associative memories (AMs) are mathematical constructs motivated by the human brain ability to store and recall information [1, 2, 3, 4]. Such as the biological neural network, an AM should be able to retrieve a memorized information from a possibly incomplete or corrupted item. An AM designed for the storage and recall of fuzzy sets is called fuzzy associative memory (FAM) [5, 6]. Precisely, a FAM is designed for the storage of associations $(A^1,B^1),(A^2,B^2),\ldots,(A^p,B^p)$, where A^ξ and B^ξ are fuzzy sets for all $\xi = 1,\ldots,p$. Afterward, the FAM model is expect to retrieve a certain B^ξ in response to the presentation of a partial or noisy version $\tilde{A}^\xi$ of A^ξ. Examples of FAM applications are pattern classification and recognition [7, 8], optimization, computer vision and image retrieval [9, 10, 11], prediction [12, 13], and control [14, 5].

Research on FAM models dates to the early 1990s with Kosko's work [5]. Generally speaking, Kosko's FAM stores an association (A^ξ,B^ξ) in a matrix M^ξ using either the correlation-minimum or the correlation-product encoding scheme. In order to avoid crosstalk, Kosko proposed a FAM bank in which the output is determined by a weighted sum of the fuzzy sets produced separately by each FAM matrix. Specifically, if X is the input fuzzy set, the output of a FAM bank is $Y = \sum_{\xi=1}^{p} w_\xi \tilde{Y}^\xi$, where $\tilde{Y}^\xi$ is given by either the max-min or max-product composition of M^ξ by X, for $\xi = 1,\ldots,p$.

The separate storage of FAM matrices partially solves the crosstalk problem, but it consumes a lot of space. Thus, many researchers developed FAM models in which p associations are encoded in a single matrix. For instance, Chung and Lee proposed an encoding scheme based on the max-T composition, where T refers to a triangular norm [15]. Similarly, the implicative fuzzy learning proposed by Sussner and Valle determines a unique FAM matrix using the min-I composition, where I denotes a residual implication [16]. The content-association associative memory (ACAM) proposed recently by Bui et al. also encodes a set of associations $\{(A^\xi,B^\xi):\xi = 1,\ldots,p\}$ using a single matrix [11]. A comprehensive review on FAM models in which the associations are encoded in a single matrix can be found in [6, 10].

Besides the active research on matrix-based FAMs, there is an increasing interest on non-distributive FAM models such as the Θ-FAMs introduced recently by Esmi et al. [8]. In general terms, a Θ-FAM yields the union $\cup_{\gamma\in\Gamma} B^\gamma$, where $\Gamma \subseteq \{1,\ldots,p\}$ is the set of the indexes that maximizes a certain function of the input fuzzy set. For instance, a SM-FAM is obtained by considering the indexes that maximizes the similarity measure between A^ξ and X.

The recurrent exponential fuzzy associative memories (RE-FAMs), previously called fuzzy exponential recurrent neural network (FERNN), also belong to the class of non-distributive models [17]. They have been derived from the multivalued exponential recurrent associative memories (MERAMs) of Chiueh and Tsai using concepts from fuzzy set theory [18]. Like MERAMs, RE-FAMs only implement autoassociative memories, that is, they are designed for the storage and recall of fuzzy sets $A^1,\ldots,A^p$. Furthermore, RE-FAMs are recurrent models: They produce a sequence of fuzzy sets $X_0,X_1,\ldots$ which presumably converges to the desired output. Indeed, we show that the output of a single-step RE-FAM can be made as close as desired to a certain convex combination of the fuzzy sets $A^1,\ldots,A^p$ by increasing the parameter (basis) of the exponential.

The paper is organized as follows. Section 2 briefly reviews the MERAMs of Chiueh and Tsai. RE-FAMs and a theoretical result concerning their recall capability are discussed in Section 3. Computational experiments concerning the retrieval of corrupted gray-scale images are given in Section 4. The paper finishes with some concluding remarks in Section 5.

2. Multivalued Exponential Recurrent Associative Memories

In the early 1990s, Chieh and Goodman introduced the class of recurrent correlation associative memories, which includes the Hopfietd network and some high storage capacity models such as the exponential correlation associative memories (ECAMs) as particular instances. Besides the very high storage capacity, ECAMs exhibit excellent error correction capabilities. On the downside, they are designed for the storage and recall of bipolar vectors.

Many applications of AMs, including the retrieval of gray-scale images in the presence of noise, require the storage and recall of real-valued vectors or fuzzy sets [5, 7, 8, 9, 10, 11, 12, 13, 14]. In 1993, Chiueh and Tsai extended ECAMs for multi-valued vectors [18]. The resulting models, called multivalued exponential recurrent associative memories (MERAMs), are defined as follows.

Let $\mathbb{K} = \{\kappa_1, \kappa_2, \ldots, \kappa_K\}$ denote a K-valued set and α be a positive real number. Consider a fundamental memory set $\{A^1, \ldots, A^p\} \subseteq \mathbb{K}^n$, where each A^ξ is a multivalued column vector. Given a multivalued input vector $X_0 \in \mathbb{K}^n$, a MERAM defines recursively the sequence of vectors $X_0, X_1, \ldots$ according to the equation

$$X_{t+1} = \frac{\sum_{\xi=1}^{p} A^\xi e^{\alpha \Psi\left(A^\xi, X_t\right)}}{\sum_{\xi=1}^{p} e^{\alpha \Psi\left(A^\xi, X_t\right)}}, \quad \forall t = 0, 1, \ldots, \tag{1}$$

where $\Psi\big(A^\xi, X_t\big)$ measures – in a broad sense – the similarity between A^ξ and X_t. For example, Ψ may refer to the direction cosine or the Euclidean distance-based similarity measure given respectively by

$$\Psi_C(A^\xi, X_t) = \frac{\langle A^\xi, X_t \rangle}{\|A^\xi\|_2 \|X_t\|_2} \quad \text{and} \quad \Psi_E(A^\xi, X_t) = \frac{1}{1 + \|A^\xi - X_t\|_2}, \tag{2}$$

where $\langle \cdot, \cdot \rangle$ denotes the usual inner product and $\|\cdot\|_2$ is the Euclidean norm.

3. Recurrent Exponential Fuzzy Associative Memories

In this section, we shall adapt the MERAMs in a rather straightforward manner for the storage and recall of fuzzy sets. Before, however, let us briefly review some well-established concepts from fuzzy set theory.

A fuzzy set A on a universe of discourse U is identified by its membership function $A : U \to [0,1]$, where $A(u)$ denotes the degree to which the element $u \in U$ belongs to the fuzzy set A. The family of all fuzzy subsets of U is denoted by $\mathscr{F}(U)$. We say that $A \in \mathscr{F}(U)$ is a subset of $B \in \mathscr{F}(U)$, and write $A \subseteq B$, if $A(u) \leq B(u)$ for all $u \in U$. Also, $\bar{A}$ denotes the standard complement of a fuzzy set A, that is, $\bar{A}(u) = 1 - A(u), \forall u \in U$.

A similarity measure, also known as equality index or fuzzy equivalence, is a function that maps pair of fuzzy sets to a number in the unit interval $[0,1]$, representing the degree to which those fuzzy sets are equal [19, 20]. Applications of similarity measures include fuzzy neural networks [8, 21], fuzzy clustering [22], and rule base simplification [23]. In the following, we consider the normalized version of the axiomatic definition provided by Xuecheng [24]:

Definition 1 (Similarity Measure) *A similarity measure is a function $\mathscr{S} : \mathscr{F}(U) \times \mathscr{F}(U) \to [0,1]$ which satisfies the following properties for any fuzzy sets $A, B, C, D \in \mathscr{F}(U)$:*

1. $\mathscr{S}(A,B) = S(B,A)$.
2. $\mathscr{S}(A,A) = 1$.
3. *If $A \subseteq B \subseteq C \subseteq D$, then $\mathscr{S}(A,D) \leq \mathscr{S}(B,C)$.*
4. *$\mathscr{S}(A,\bar{A}) = 0$, for every crisp set $A \in \mathscr{P}(U)$.*

In addition, we say that $\mathscr{S}$ is a strong similarity measure if $\mathscr{S}(A,B) = 1$ implies $A = B$.

Example 1 *Let $U = \{u_1, \cdots, u_n\}$ be a finite universe of discourse. The following presents three instances of strong similarity measures [19, 25, 26, 27].*

1. *Gregson similarity measure:*
$$\mathscr{S}_G(A,B) = \frac{\sum_{j=1}^{n} A(u_j) \wedge B(u_j)}{\sum_{j=1}^{n} A(u_j) \vee B(u_j)}, \tag{3}$$
where the symbols "$\wedge$" and "$\vee$" denote respectively the minimum and maximum operations.

2. *Eisler and Ekman similarity measure:*
$$\mathscr{S}_E(A,B) = \frac{2\sum_{j=1}^{n} A(u_j) \wedge B(u_j)}{\sum_{j=1}^{n} A(u_j) + \sum_{j=1}^{n} B(u_j)}. \tag{4}$$

3. *Complement of the relative Hamming distance:*
$$\mathscr{S}_H(A,B) = 1 - \frac{1}{n}\sum_{j=1}^{n} |A(u_j) - B(u_j)|. \tag{5}$$

A *recurrent exponential fuzzy associative memory* (RE-FAM) is a two-layer dynamic neural network designed for the storage of a family $\mathscr{A} = \{A^1, A^2, \cdots, A^p\} \subseteq \mathscr{F}(U)$ of fuzzy sets [17]. The first layer computes an exponential of the similarity between A^ξ and the current state, represented by a fuzzy set $X_t \in \mathscr{F}(U)$, for each $\xi \in \{1, \ldots, p\}$. The output layer yields a convex combination of $A^1, \ldots, A^p$ whose weights are the outputs of the previous layer. Figure 1 shows a block diagram of a RE-FAM. Formally, a RE-FAM is defined as follows:

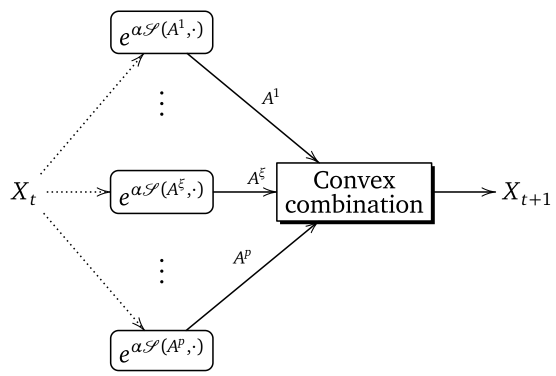


Figure 1. Block diagram of a RE-FAM.

Definition 2 (RE-FAM) *Consider a real number $\alpha > 0$, fuzzy sets $A^1,\ldots,A^p \in \mathscr{F}(U)$, and let $\mathscr{S} : \mathscr{F}(U)\times\mathscr{F}(U) \to [0,1]$ denote a fuzzy similarity measure. Given a fuzzy set $X_0 \in \mathscr{F}(U)$, a RE-FAM produces a sequence $\{X_t\}$ of fuzzy sets according to the following evolution equation for all $u \in U$:*

$$X_{t+1}(u) = \frac{\sum_{\xi=1}^{p} A^{\xi}(u)e^{\alpha\mathscr{S}(A^{\xi},X_t)}}{\sum_{\eta=1}^{p} e^{\alpha\mathscr{S}(A^{\eta},X_t)}}, \quad \forall t = 0,1,\ldots. \tag{6}$$

The following theorem shows that the output of a single-step RE-FAM converges point-wise to a convex combination of certain fundamental memories as the parameter $\alpha > 0$ tends to infinity. In other words, the fuzzy set X_1 can be made as close as desired to a certain convex combination of $A^1,\ldots,A^p$ by choosing a sufficiently large parameter $\alpha > 0$.

Theorem 1 *Consider a family of fuzzy sets $\{A^1,\cdots,A^p\} \subseteq \mathscr{F}(U)$ and let $\mathscr{S}$ denote a similarity measure. Given a fuzzy set $X_0 \in \mathscr{F}(U)$, let $\Gamma \subseteq \{1,\ldots,p\}$ be the set of the indexes of $A^1,\ldots,A^p$ with the largest similarity degree with X_0, that is,*

$$\Gamma = \{\gamma : \mathscr{S}(A^{\gamma},X_0) \geq \mathscr{S}(A^{\xi},X_0), \forall \xi = 1,\ldots,p\}. \tag{7}$$

If $X_1 \in \mathscr{F}(U)$ denotes the output of a single-step RE-FAM given by (6) with $t = 0$, then

$$\lim_{\alpha\to\infty} X_1(u) = \frac{1}{\mathrm{Card}(\Gamma)}\sum_{\xi\in\Gamma} A^{\xi}(u), \quad \forall u \in U. \tag{8}$$

Proof. Let $\upsilon = \max_{\xi=1:p}\{\mathscr{S}(A^{\xi},X_0)\}$ denote the largest similarity degree between the input fuzzy set X_0 and $A^1,\ldots,A^p$. From (7), we have $\upsilon = \mathscr{S}(A^{\gamma},X_0)$ for all $\gamma \in \Gamma$ while $\mathscr{S}(A^{\xi},X_0) < \upsilon$ if $\xi \notin \Gamma$. Now, multiplying both numerator and denominator of (6) by $e^{-\alpha\upsilon}$ and breaking up the sums, we obtain

$$\begin{aligned} X_1(u) =& \frac{\sum_{\xi=1}^{p} A^{\xi}(u)e^{\alpha(\mathscr{S}(A^{\xi},X_0)-\upsilon)}}{\sum_{\eta=1}^{p} e^{\alpha(\mathscr{S}(A^{\eta},X_0)-\upsilon)}} \\ =& \frac{\sum_{\xi\in\Gamma} A^{\xi}(u) + \sum_{\xi\notin\Gamma} A^{\xi}(u)e^{\alpha(\mathscr{S}(A^{\xi},X_0)-\upsilon)}}{\sum_{\eta\in\Gamma} 1 + \sum_{\eta\notin\Gamma} e^{\alpha(\mathscr{S}(A^{\eta},X_0)-\upsilon)}}. \end{aligned} \tag{9}$$

Since $\mathscr{S}(A^{\eta},X_0)-\upsilon < 0$ for all $\eta \notin \Gamma$, the second sum in both numerator and denominator vanishes as $\alpha \to \infty$. Hence,

$$\lim_{\alpha\to\infty} X_1(u) = \frac{\sum_{\xi\in\Gamma} A^{\xi}(u)}{\sum_{\eta\in\Gamma} 1}, \quad \forall u \in U, \tag{10}$$

which is exactly the identity given by (8).

From Theorem 1, if $\mathscr{S}(A^{\gamma},X_0) > \mathscr{S}(A^{\xi},X_0)$ for all $\xi \neq \gamma$, then the output of a single-step RE-FAM converges point-wise to A^{γ} as $\alpha \to \infty$. Hence, we conjecture that the basin of attraction of A^{γ} is the region

$$\mathscr{R}^{\gamma} = \{X \in \mathscr{F}(U) : \mathscr{S}(A^{\gamma},X) > \mathscr{S}(A^{\xi},X), \forall \xi \neq \gamma\}, \tag{11}$$

if α is sufficiently large. In other words, $\mathscr{R}^{\gamma}$ corresponds to the family of fuzzy sets which are more similar to A^{γ} than any other fundamental memory A^{ξ}, $\xi \neq \gamma$. Furthermore, if $\mathscr{S}$ is a strong similarity measure, then $A^{\gamma} \in \mathscr{R}^{\gamma}$ because $\mathscr{S}(A^{\gamma},A^{\gamma}) > \mathscr{S}(A^{\gamma},A^{\xi})$ for all $\xi \neq \gamma$. In other words, a RE-FAM exhibits optimal absolute storage capacity if $\mathscr{S}$ is a strong similarity measure and the parameter α is sufficiently large.

Remark 1 *The pattern recalled by an autoassociative similarity measure FAM (SM-FAM) of Esmi et al. under presentation of $X \in \mathscr{F}(U)$ is the fuzzy set*

$$Y = \bigcup_{\gamma\in\Gamma} A^{\gamma}, \tag{12}$$

where Γ is the set of indexes given by (7) [8]. Hence, an autoassociative SM-FAM differs from a single-step RE-FAM with a large parameter α in the way A^{γ}, $\gamma \in \Gamma$, are combined to produce the output. Furthermore, both FAM models probably coincide if Γ has an unique element and α is sufficiently large.

4. Computational Experiments

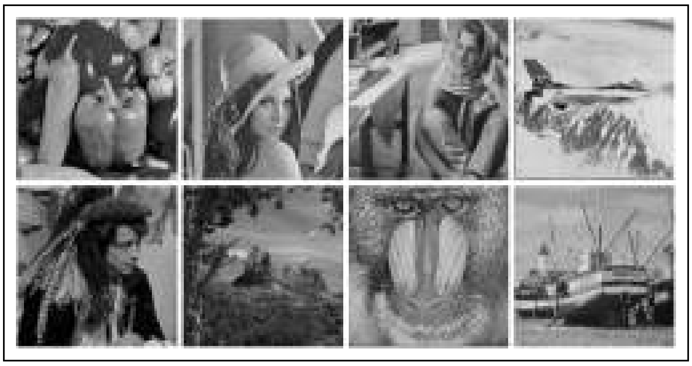


Figure 2. Original gray-scale images of size 128×128 **and** 256 **gray-levels.**

Let us perform some experiments concerning the retrieval of corrupted gray-scale images. Consider the eight images displayed in Figure 2. Each of these images corresponds to a fuzzy set $A^{\xi} \in \mathscr{F}(U)$, $U = \{1,\ldots,128\} \times \{1,\ldots,128\}$ [26]. First, we synthesized RE-FAMs designed for the storage of these eight gray-scale images by considering the parameter $\alpha = 30$ and the similarity measures given by (3), (4), and (5). For comparison purposes, we also stored the eight gray-scale images in the following FAM models: Lukasiewicz implicative fuzzy associative memory (IFAM) [16], the content-association associative memory (ACAM) with the parameter $\eta = 2$ [11], and the SM-FAM based on the three similarity measures given in Example 1 [7, 8]. In addition, we confronted the FAM models with the MERAMs based on Ψ_C and Ψ_E as well as the optimal linear associative memory (OLAM) [2], the kernel associative memory (KAM) [28], the complex-sigmoid Hopfield network (CSIGM) [29], and a certain subspace projection autoassociative memory (SPAM) [30].

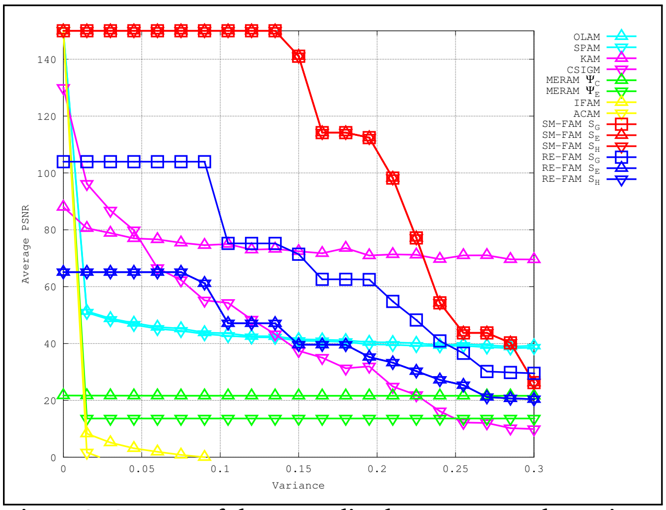


Figure 3. Average of the normalized error versus the variance of the Gaussian noise.

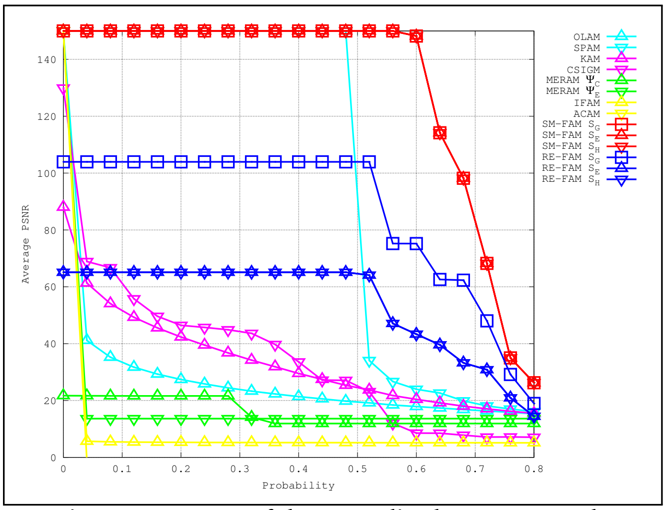


Figure 4. Average of the normalized error versus the probability of salt and pepper noise.

Subsequently, we probed each memory model with gray-scale images corrupted by Gaussian noise with zero mean and variance ranging from 0 to 0.3. We also probed the AMs with images corrupted by salt and pepper noise with densities varying from 0 to 0.8. In the computational experiments, we iterated (6) until either $\|X_{t+1} - X_t\|_2 \leq 10^{-6}$ or $t > 20$, where $\|\cdot\|_2$ denotes the usual Euclidean norm [27]. Figures 3 and 4 show the peak signal-to-noise ratio (PSNR) rates produced by the memory models averaged in 80 experiments, that is, each original image was distorted 10 times for a certain noise intensity. Specifically, we plotted the arithmetic mean

$$\frac{1}{80}\sum_{\mu=1}^{10}\sum_{\xi=1}^{8} 150 \wedge \left[20\log\left(\frac{\sqrt{n}}{\|A^\xi - Y^{\xi\mu}\|_2}\right)\right], \qquad (13)$$

where $Y^{\xi\mu}$ denotes the output of a certain AM under presentation of a corrupted version $\tilde{A}^{\xi\mu}$ of A^ξ at the μ-th experiment (or simulation), by the noise intensity introduced into $\tilde{A}^{\xi\mu}$. We would like to point out that we bounded the PSNR value above by 150 because it tends to infinity as $\|A^\xi - Y^{\xi\mu}\|_2 \to 0$.

Except the MERAM based on Ψ_C, all AM models yielded large PSNR rates under presentation of an undistorted input, that is, when the variance or probability equals to zero. In spite of the optimal absolute storage capacity, both IFAM and ACAM are unable to cope with patterns corrupted by mixed noise such as Gaussian noise or salt and pepper noise. In fact, these two FAM models exhibit an excellent tolerance with respect to negative noise, that is, they are able to retrieve A^ξ only if the input fuzzy set X satisfies $X(u) \leq A^\xi(u)$ for some $u \in \mathscr{F}(U)$ [11, 16].

Note that the SM-FAM yielded the largest PSNR rates. The second largest PSNR rates have been produced by the RE-FAM based on $\mathscr{S}_G$ for salt and pepper noise and Gaussian noise with $\sigma^2 < 0.15$. However, the difference between these two models reduce by increasing the parameter α of the RE-FAM.

Finally, Figures 5 and 6 provide a visual interpretation of images retrieved by the AM models. Precisely, Figure 5 depicts the image "peppers" corrupted by Gaussian noise with variance $\sigma^2 = 0.08$ followed by the images retrieved by the AMs. Similarly, Figure 6 shows the image "lena" corrupted by salt and pepper noise with probability $p = 0.1$ followed by the retrieved images. Observe that the IFAM and ACAM models produced whiter images as output. The MERAM based on Ψ_E failed to retrieve the original images. The MERAM based on Ψ_C retrieved the original "peppers" image but failed to recall the "lena". The other AM models retrieved images visually similar to the undistorted images.

5. Concluding Remarks

In this paper, we investigated the fuzzy exponential recurrent neural networks (RE-FAMs), which can be used for the storage and recall of fuzzy sets. In contrast to many FAM models, a RE-FAM is described by a two-layer recurrent fuzzy neural network. The nodes in the first layer compute an exponential of the fuzzy similarity measure between the current state and the stored fuzzy sets. The next fuzzy set is obtained by a weighted average of the stored fuzzy sets. We showed in this paper that, when $\alpha \to \infty$, the output of a single-step RE-FAM converges to a linear combination of the fuzzy sets that are the most similar to the input.

In this paper, we also observed that the RE-FAMs may produce excellent results for the retrieval of gray-scale images corrupted by either Gaussian noise or salt and pepper noise. In particular, they outperformed the multivalued exponential recurrent associative memory (MERAM) based on Ψ_C e Ψ_E given by (2). Also, the largest PSNR rates have been obtained by the similarity measure fuzzy associative memory (SM-FAM) models of Esmi et al. Notwithstanding, by increasing the value of the parameter α of the RE-FAM, we can obtain results similar to the those produced by the SM-FAMs models.

In the future, we plan investigate further the convergence of the sequence produced by an RE-FAM. We also intent to study the effect of the fuzzy similarity and other measures on the storage capacity as well as the noise tolerance of RE-FAMs. Finally, the relationship between the RE-FAMs and other fuzzy AM models, including the SM-FAMs and the KAM model, require further attention.

Acknowledgments. This work was supported in part by CAPES, CNPq under grant no. 305486/2014-4, and FAPESP under grant no. 2013/12310-4.

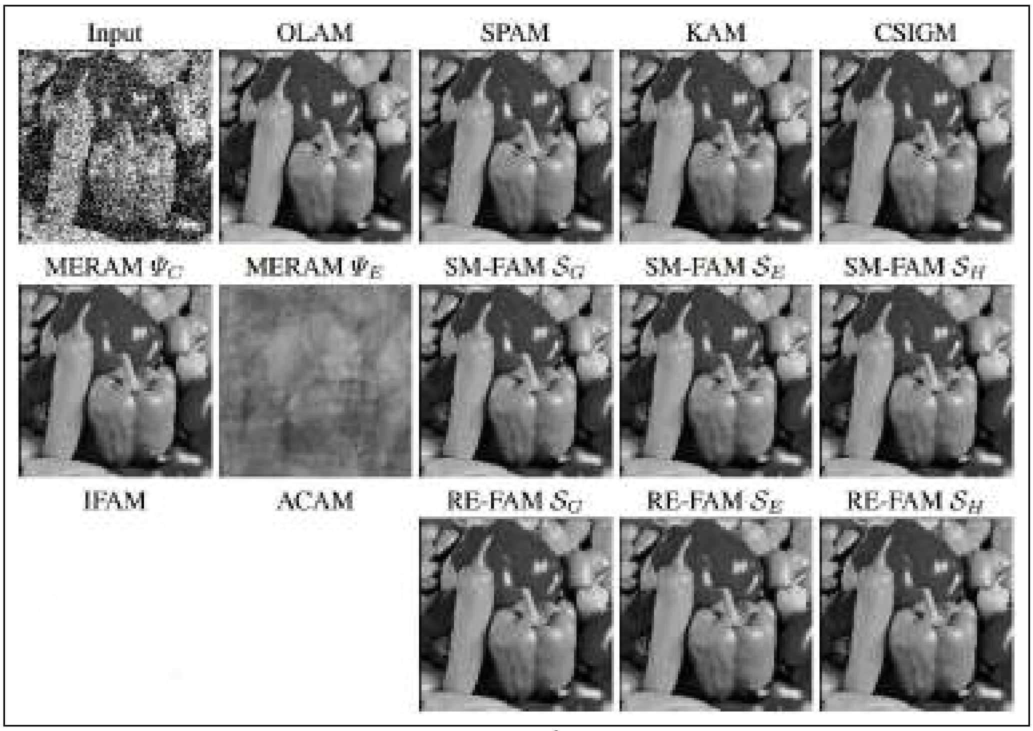


Figure 5. Input image corrupted by Gaussian noise with variance $\sigma^2 = 0.08$ followed by the images retrieved by the AM models.

Figure 6. Input image corrupted by salt and pepper noise with probability $p = 0.1$ followed by the images retrieved by the memory models.

References

[1] Austin, J.: Associative Memory. In Fiesler, E., Beale, R., eds.: Handbook of Neural Computation. Oxford University Press (1997) F1.4:1–F1.4:7

[2] Kohonen, T.: Self-organization and associative memory. 2rd edition edn. Springer-Verlag New York, Inc., New York, NY, USA (1987)

[3] Hassoun, M.H., ed.: Associative Neural Memories: Theory and Implementation. Oxford University Press, Oxford, U.K. (1993)

[4] Hassoun, M.H., Watta, P.B.: Associative Memory Networks. In Fiesler, E., Beale, R., eds.: Handbook of Neural Computation. Oxford University Press (1997) C1.3:1–C1.3:14

[5] Kosko, B.: Neural Networks and Fuzzy Systems: A Dynamical Systems Approach to Machine Intelligence. Prentice Hall, Englewood Cliffs, NJ (1992)

[6] Sussner, P., Valle, M.E.: Fuzzy Associative Memories and Their Relationship to Mathematical Morphology. In Pedrycz, W., Skowron, A., Kreinovich, V., eds.: Handbook of Granular Computing. John Wiley and Sons, Inc., New York (2008) 733–754

[7] Esmi, E.L., Sussner, P., Valle, M.E., Sakuray, F., Barros, L.: Fuzzy Associative Memories Based on Subsethood and Similarity Measures with Applications to Speaker Identification. In: Lecture Notes in Computer Science: International Conference on Hybrid Artificial Intelligence Systems (HAIS 2012). Springer-Verlag Berlin Heidelberg, Berlin, Germany (2012) 479–490

[8] Esmi, E., Sussner, P., Bustince Sola, H., Fernandez, J.: Θ-fuzzy associative memories (Θ-FAMs). IEEE Transactions on Fuzzy Systems **23**(2) (2015) 313 – 326

[9] Sussner, P., Esmi, E.L., Villaverde, I., Graña, M.: The Kosko Subsethood Fuzzy Associative Memory (KS-FAM): Mathematical Background and Applications in Computer Vision. Journal of Mathematical Imaging and Vision **42**(2–3) (February 2012) 134–149

[10] Binh, P., Hoa, N.: Compare effective fuzzy associative memories for grey-scale image recognition. In Vinh, P., Hung, N., Tung, N., Suzuki, J., eds.: Context-Aware Systems and Applications. Volume 109 of Lecture Notes of the Institute for Computer Sciences, Social Informatics and Telecommunications Engineering. Springer Berlin Heidelberg (2013) 258–267

[11] Bui, T.D., Nong, T.H., Dang, T.K.: Improving learning rule for fuzzy associative memory with combination of content and association. Neurocomputing **149A**(0) (2015) 59 – 64

[12] Sussner, P., Valle, M.E.: Recall of Patterns Using Morphological and Certain Fuzzy Morphological Associative Memories. In: Proceedings of the IEEE World Conference on Computational Intelligence 2006, Vancouver, Canada (2006) 209–216

[13] Sussner, P., Miyasaki, R., Valle, M.E.: An Introduction to Parameterized IFAM Models with Applications in Prediction. In: Proceedings of the 2009 IFSA World Congress and 2009 EUSFLAT Conference, Lisbon, Portugal (July 2009) 247–252

[14] Kong, S.G., Kosko, B.: Adaptive Fuzzy Systems for Backing up a Truck-and-Trailer. IEEE Transactions on Neural Networks **3**(2) (1992) 211–223

[15] Chung, F., Lee, T.: On Fuzzy Associative Memory with Multiple-Rule Storage Capacity. IEEE Transactions on Fuzzy Systems **4**(3) (1996) 375–384

[16] Sussner, P., Valle, M.E.: Implicative Fuzzy Associative Memories. IEEE Transactions on Fuzzy Systems **14**(6) (December 2006) 793–807

[17] Valle, M.E.: Fuzzy Exponential Recurrent Neural Networks for Gray-scale Image Retrieval. In: Proceedings of the Third Brazilian Conference on Fuzzy Systems (III CBSF), João Pessoa, Brazil (August 2014)

[18] Chiueh, T.D., Tsai, H.K.: Multivalued associative memories based on recurrent networks. IEEE Transactions on Neural Networks **4**(2) (March 1993) 364–366

[19] Dubois, D., Prade, H.: Fuzzy sets and systems: theory and applications. Academic Press, New York (1980)

[20] Klir, G.J., Yuan, B.: Fuzzy Sets and Fuzzy Logic: Theory and Applications. Prentice Hall, Upper Saddle River, NY (1995)

[21] Ballini, R., Gomide, F.: Equality index and learning in recurrent fuzzy neural networks. In: Proceedings of the 12th IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2003). Volume 1., St Louis, USA (May 2003) 155–160

[22] Yager, R.R., Filev, D.P.: Summarizing data using a similarity based mountain method. Information Sciences **178**(3) (February 2008) 816–826

[23] Setnes, M., Babuska, R., Kaymak, U., van Nauta Lemke, H.: Similarity measures in fuzzy rule base simplification. Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on **28**(3) (Jun 1998)

[24] Xuecheng, L.: Entropy, distance measure and similarity measure of fuzzy sets and their relations. Fuzzy Sets and Systems **52**(3) (1992) 305–318

[25] Zwick, R., Caristein, E., Budescu, D.V.: Measures of Similarity Among Fuzzy Concepts: A Comparative Analysis. International Journal of Approximate Reasoning **1**(2) (April 1987) 221–242

[26] Lowen, R., Peeters, W.: Distances between fuzzy sets representing grey level images. Fuzzy Sets and Systems **99**(2) (1998) 135 – 149

[27] Bloch, I.: On fuzzy distances and their use in image processing under imprecision. Pattern Recognition **32**(11) (1999) 1873 – 1895

[28] Zhang, B.L., Zhang, H., Ge, S.S.: Face Recognition by Applying Wavelet Subband Representation and Kernel Associative Memory. IEEE Transactions on Neural Networks **15**(1) (January 2004) 166–177

[29] Tanaka, G., Aihara, K.: Complex-Valued Multistate Associative Memory With Nonlinear Multilevel Functions for Gray-Level Image Reconstruction. IEEE Transactions on Neural Networks **20**(9) (September 2009) 1463–1473

[30] Valle, M.: A Robust Subspace Projection Autoassociative Memory Based on the M-Estimation Method. IEEE Transactions on Neural Networks and Learning Systems **25**(7) (July 2014) 1372–1377

SCIENTIFIC REPORT

Adaptation of the Notification Oriented Paradigm (NOP) for the Development of Fuzzy Systems

Luiz Carlos Viana Melo and Jean Marcelo Simão* and João Alberto Fabro
Graduation Program on Applied Computing (PPGCA)
Graduation Program on Electrical Engineering and Computer Science (CPGEI)*
Federal University of Technology - Paraná (UTFPR)
Av. Sete de Setembro, 3165 – 80230-901 – Curitiba – PR – Brazil
http://ppgca.ct.utfpr.edu.br/

Abstract. The Notification Oriented Paradigm (NOP) combines and evolves the event based programming with the declarative programming in order to solve some problems of both paradigms. Breaking down one application into a network of smaller computational entities, such as logic-causal and factual notifier entities that processes only when needed, the NOP eliminates the need to perform unnecessary computations and improves the code reusability. Fuzzy systems, in turn, perform inference based on knowledge bases (IF-THEN rules) that can cope with problems involving imprecision. Since NOP uses IF-THEN rules in an alternative way, by reducing evaluation redundancy and coupling, this research was conducted to identify, propose and evaluate the needed changes on the NOP to allow its use in the development of fuzzy systems. The tests results showed a significant reduction in the number of evaluated rules, which may represent improvement in performance of the applications.

Keywords. Notification Oriented Paradigm, Fuzzy Systems, Rule Based Systems.

1. Introduction

The Notification Oriented Paradigm (NOP) was proposed by Simão [7] as a control solution that later evolved into a programming paradigm. NOP sought inspiration from both imperative and declarative paradigms, more precisely from the object oriented and logic sub-paradigms, at same time that it aims to solve some of their deficiencies [10].

NOP uses the main advantages of the declarative paradigm, namely the expressiveness of causal rules from rule based systems, which has abstraction and language closer to the form of human cognition. It also uses the advantages of the imperative paradigm, namely the code reusability, flexibility and abstraction through classes and objects from the object oriented sub-paradigm [10]. However, NOP presents a new form of inference, which differs from current paradigms.

The main idea behind NOP is the way the software detects changes in variables and makes inferences about it. In the current imperative paradigms, there are two forms to detect changes in values of a variable: through polling or event notification. In polling, the program loop performs successive evaluations of the system variables, performing some logic evaluations on the values and triggering actions when certain conditions are met. This approach is considered sequential because only one condition is checked at a time in a given thread. Due to the fact that the loop is executed even when the variables do not change their values, this approach wastes computing resources [4].

An alternative to polling is event oriented programming. In this approach, any processing depends on events. Events can be triggered by user's actions or other situations that can provoke changes in the internal state of the program. In some approaches this eliminates the need to have a loop that checks the states of variables, which reduces the unnecessary computing. However, this alternative makes the application development usually more complex, resulting in bigger programs. In addition, given the hardware constraints, the event controller system may perform polling to dispatch events [4].

In the case of declarative programming, the programmer must focus on what a program should accomplish instead of how it should be accomplished. This frees the programmer from handling many unimportant details. However, they are usually slower to execute and less flexible than imperative programming [3, 4].

Having these problems as motivation, NOP combines and evolves ideas of both event based and the declarative programming in order to solve them. Actually, NOP eliminates the need to perform unnecessary computing and enhance modular decoupling, thereby facilitating code reusability for example. NOP achieves this by breaking down one application into a network of smaller computational notifier entities that are executed only when needed [14].

Besides, in the past, some preliminary studies were conducted to evaluate if it would be possible to extend the NOP paradigm to fuzzy systems [12, 13]. These studies were motivated due to the fact that in both NOP and fuzzy systems the knowledge can be described in the IF-THEN rules format such as those used in natural language. Those preliminary studies used as case studies applications in the field of robotics.

The results of the previous studies on fuzzy NOP demonstrated a significant reduction in the number of evaluated rules, which could improve the application performance.

However, they did not describe in details the changes performed, which makes difficult to replicate them in others materialization of the paradigm. Consequently, this ended up causing the lack of support to the development of fuzzy systems by current implementations of NOP. Also the results in terms of processing time were not good due to the style of materializations of NOP at the time, i.e. frameworks in C++ with expensive data structures.

In addition, many works on fuzzy systems use embedded systems to implement their concepts [5, 6]. Due to the restrictions of processing, memory and energy consumption, it would be interesting to find a way to optimize these factors. NOP can be an alternative to this since it solves these problems through the use of passive entities that will be processed only when necessary.

In the following sections, this paper presents the related concepts of NOP and fuzzy systems, the modifications performed on the paradigm to provide fuzzy concepts and inference, and a simple application, to evaluate its applicability.

2. Notification Oriented Paradigm (NOP)

This paradigm basis was first proposed as a control mechanism to supply the needs related to modern production systems [10]. Later, the author realized that this model could be applied on many problem domains. Therefore, he proposed and adjusted the mechanism to provide a general solution for discrete control. In addition, he also realized that the model could be used to guide programmers in the conception of applications, resulting in the so-called Notification Oriented Paradigm (NOP) [3]. This paradigm is detailed in the following subsections.

2.1. Paradigm structure

NOP introduces a set of new concepts that can be applied to design, build and execute computer programs. The main concept is the use of small, active and decoupled entities that collaborates by means of notifications in order to perform the logical and causal calculation existing in the software [8, 10]. The knowledge in the paradigm is represented by causal rules and factual base elements that are naturally understood by programmers of the current paradigms [10].

The entities that compose the paradigm are as follows:

- *Fact Base Element* (FBE): is an entity of the observed system. It aggregates the *Attributes* that represents the facts about the cited entity and the *Methods* that allow for the execution of functionalities associated to this element;

- *Attribute*: is a value of the FBE that represents one of its features, composing its state;

- *Premise*: is a logical inquiry between an *Attribute* (that belong to some FBE) and a value (for instance, “Is *X* equal to 2 ?"). A *Premise* is composed of an *Attribute*, a comparison operator, and a third element, that can be a constant value, or another *Attribute*;

- *Condition*: is a logical relationship between the *Rule's Premises*. This relationship is usually denoted by the conjunction (*AND*) and disjunction (*OR*) logical connectors, or a combination of both;

- *Rule*: is a rule in the system's rule set; this entity associates a *Condition* to an *Action*. The relationship implemented by this entity is casual implication, which can be read as “IF the *Condition* is active (antecedent), THEN activate the *Action* (consequent)";

- *Action*: is an action to be executed when the associated *Rule* was been approved. This element contains a set of *Instigations* that must be processed;

- *Instigation*: is the activity that induces the execution of a *Method* over a FBE;

- *Method*: is a FBE's method, that can perform changes over *Attributes*.

Figure 1 displays the class diagram of the cited entities and the relationship among them.

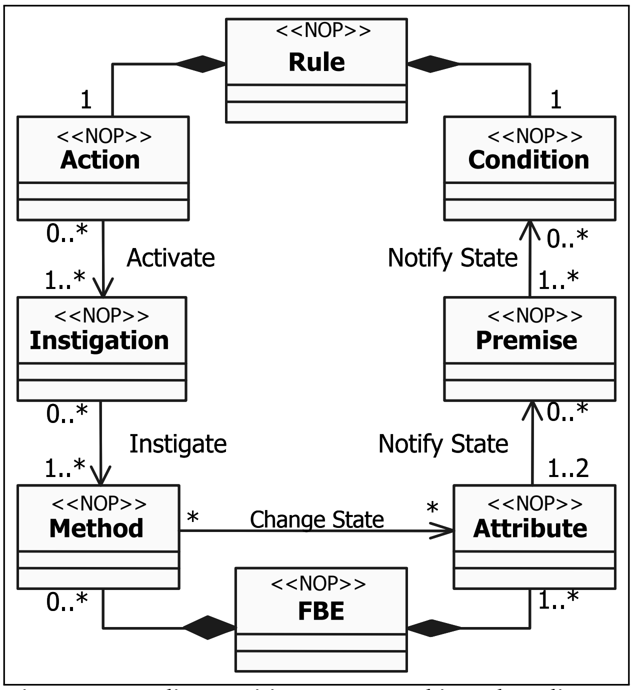


Figure 1. Paradigm entities represented in a class diagram [10].

The collaboration between the entities is the basis for the paradigm inference mechanism, and the collaboration between them is done through an exchange of punctual notifications. The following section shows the paradigm notification mechanism by displaying the role of each entity.

2.2. NOP notification engine

The notification engine is the internal process to execute the NOP instances, which determines the application execution flow. Through this mechanism, the program tasks are split among the entities, which cooperate through notifications telling each other about the share of their contributions to form the application execution flow [3]. Figure 2 shows an example of the notification chain.

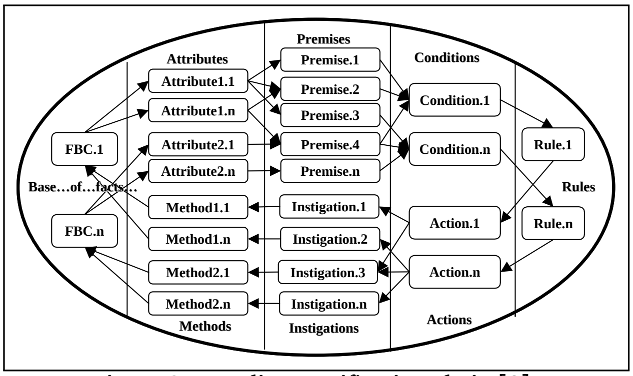


Figure 2. Paradigm notification chain [8].

The inference process starts when the state (value) of an FBE's *Attribute* is changed (e.g. FBC.1 and FBC.n, seen in figure 2, left side). Each change causes the *Attribute* to notify the interested *Premises*. The *Premises*, after receiving the notification, perform logical calculation to determine if their state have to change (from *false* to *true*, for example). This calculation is done by comparing the *Attribute* value with the *Premise* value using a comparison operator.

Using the same notification procedure, *Premises* that have changed their logical values notify the interested *Conditions*. Upon receiving the *Premises* notifications, the *Conditions* do their logical calculation from a causal logical expression. Each *Conditions* defines the relationship between the notifying *Premises* by the use of logical connectives ("AND" and "OR"). An expression can be read as the following example: "Premise1 activated 'AND' Premise2 activated". If the *Condition* state changes, it will notify the *Rule* to which it is associated.

The *Rule*, after receiving a notification from the *Condition*, will check if the associated *Condition* is not in conflict[3]. If this is the case, the *Rule* will notify the associated *Action*. An *Action* will perform a number of instigations through the *Instigation* entities within it. In the last step of the notification cycle, the *Instigations* will cause the FBE *Methods* to be executed, which can alter the FBE *Attributes* values, starting a new cycle of Notification Oriented Inference [16].

Through this notification engine the NOP entities will only execute after receiving a notification from another entity, which may results in resources saving and speed up in application performance [3, 10].

In the next section, a brief introduction of the main concepts of fuzzy systems is presented. In the following sections, the adaptation of the NOP paradigm to handle with fuzzy concepts and inference mechanisms is proposed and detailed.

3. Fuzzy systems

The studies on fuzzy systems started in 1965 when Zadeh [1] proposed his fuzzy sets theory. This theory generalizes the classical set theory [15], allowing the representation of concepts which cannot be represented using well defined (or crisp) limits.

A fuzzy set is the one containing the elements of the universe of discourse which has variable membership degree to this set. The equation $\mu_A(x) \in [0,1]$ represents the mathematical notation of the membership degree μ_A of an element x in a set A, where the value "1.0" indicates the complete compatibility of an element with the concept represented by the set, and the value "0.0" indicates there isn't any compatibility of the element with the set.

Another concept related with fuzzy sets is the linguistic variable, which represents an identifier that can assume one of several values. Thus, a linguistic variable can take a linguist value which represents a fuzzy set.

Fuzzy logic can be used to develop control systems that deal with imprecise information. Through a set of rules, it is possible to perform inferences that will be used in the decision making process of a system. In the most practical applications the input data is composed of values provided by sensors, that result from measurements and observations. For the use of such data by fuzzy inference systems, it is required to perform a mapping of those measurements into relevant activation of fuzzy sets [11]. This mapping is called "fuzzyfication" and results in a value of membership degree of the data to each fuzzy set.

The generated sets are used to perform the fuzzy inference. The inference step is processed through the comparison of the rules stored in the rule database. This process results in the calculation of the activation level of each rule. After the calculations the results are spread through the fuzzy rules, producing fuzzy sets that represent the consequents of each rule. The contribution of each rule (fuzzy set) is taken into account through, for instance, the union of the fuzzy sets. Beyond this point the user already has the result of the system in the format of fuzzy sets.

Once the output fuzzy set is obtained by the inference process, in the "defuzzyfication" stage it is converted into a crisp value using a defuzzyfication function like the centroid function. This is necessary because usually in practical applications exact outputs are required, not fuzzy ones [11].

The following section details the changes made in the NOP implementation to perform inferences based in fuzzy rules and concepts. Since originally NOP accepts only "crisp" activations (i.e. each NOP entity can have only one of two activation values, "true" or "false"), several modifications were necessary to allow the propagation of fuzzy activations.

4. Changes made over NOP to provide fuzzy inference capabilities

Several modifications were required in the paradigm implementation in order to allow fuzzy logic systems development. The proposed modifications where applied over the current implementation of the NOP Framework in C++ language, as presented in [9], and draw inspiration from the first fuzzy implementation developed in previous work [12].

4.1. Changes in the representation of logical state of the entities

Paradigm entities like *Premises*, *Conditions* and *Rules* have each one a logical state that is spread through notifications. This logical state has, in the originally proposed paradigm, only two possibles values: *true*(active) or *false* (inactive), represented by the integer values in $\{0,1\}$. The value 0

[3] There is mechanisms to solve conflicts as detailed in [18] and [17]

(zero) indicates that the entity is not active, and 1 (one) indicates that it is active.

To extend the representation to support fuzzy logic, the mentioned entities must represent the logical state by their activation level. The activation level can be represented as real values in the range $[0, 1]$, representing the membership degree as shown in section 2. The notification process is started when the activation level of any entity changes. These changes to the original NOP implementation could only be from inactive to active, or vice-versa, but with the extension to fuzzy activations, any change in the value can start the notification. This activation level is calculated differently for each entity as described in the following sub-sections.

4.2. Linguistic variable representation

The linguistic variable was implemented as an extension of an NOP *Attribute*, as shown in figure 3. This new element derived from the *Attribute* by C++ class inheritance has a list of fuzzy sets that represents the linguistic terms of the variable. Each set is represented by a membership function used to calculate the membership degree of a value to that set.

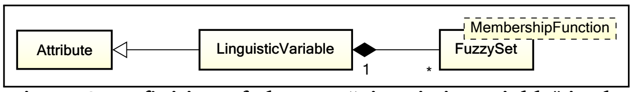


Figure 3. Definition of element "Linguistic Variable" in the NOP paradigm.

By assigning a value to this variable, it will behave as an *Attribute* and will notify the *Premises* if the assigned value is different from current value. The membership degree calculation for a given fuzzy set performed by each *Premise* as described in the section 4.3.

4.3. Changes in the calculation of the *Premises* logical state

Due the changes described in section 4.1, the *Premise* logical state is now represented by activation levels. This calculation depends on the *Premise* type. The *Premises* types are:

- Common (or crisp): *Premise* in which all logical operations are performed on non-fuzzy *Attributes*. The result of this operation will determine the *Premise* activation ("true" or "false" , represented by either 0.0 or 1.0);

- Fuzzy *Premise*: *Premise* that calculates the membership degree of the *Attribute* value in the fuzzy set associated with that *Premise*. The calculated membership degree defines the *Premise* activation level (a real number between 0.0 and 1.0).

In the case of fuzzy *Premise* the activation level will be the result of the calculation of the membership degree for the value stored in the *Attribute* to the fuzzy set referred by the *Premise*.

4.4. Readjustment of the *Condition* logical expression

The change in the *Condition* consisted in the creation of a model that represent the *Conditions* causal expressions. The model is presented in figure 4.

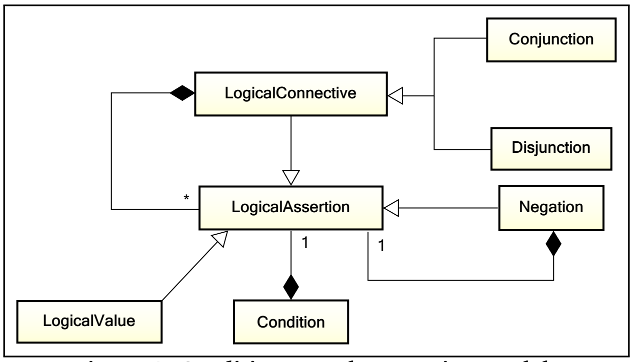


Figure 4. *Condition* casual expression model.

In this model the *Premise* is associated to an expression with a logical value. This value is updated when the *Premise* notifies the *Condition*, and the activation level of the *Premise* are then used as input for the operations with the logical connectives defined in the model. The notification also starts the calculation of the activation level of this *Condition*.

The calculation is performed using the definitions of the operations defined by Mamdani in [2] as follows:

- Intersection ($\mu_{(A\cap B)}(x) = min[\mu_A(x), \mu_B(x)]$) : logical connective **AND**;

- Union ($\mu_{(A\cup B)}(x) = max[\mu_A(x), \mu_B(x)]$) : logical connective **OR**;

- Complement ($\mu_{(\bar{A})}(x) = 1 - \mu_A(x)$) : negation logic operation.

The use of the mentioned operations allows the characteristics of the boolean logic to be "emulated", allowing for the use of *Rules* with both fuzzy and non-fuzzy *Premises*.

4.5. Changes in the active part of the inference process

With the changes mentioned in section 4.1 a *Rule* activates when there is a change in the *Condition* state. This also reflects in the *Instigations* since they are activated along the *Rules*.

In this work, it was defined that the *Instigations* now have two forms: the common *Instigation* and the fuzzy *Instigation*. The common *Instigation* represents the current definition of the paradigm *Instigation* and it maintains the current paradigm behavior. These will only activate when the *Rule* activation level is equal to one, otherwise the *Instigation* is not activated.

The fuzzy *Instigation* is activated at every change in the *Rule* activation level which it is related to. This activation executes the *Methods* associated to it, fuzzy or not. If the fuzzy *Instigation* involves the execution of a fuzzy *Method*, the *Rule* activation level is passed to it.

The fuzzy *Method* was created to represent the consequent of fuzzy logic rules. It associates the *Attribute* to the fuzzy set and performs the "defuzzyfication" operation. The operation result can be assigned to an *Attribute*, which can then trigger a new notification cycle.

With those changes, NOP can now allow the use of fuzzy versions of its entities, providing all of the necessary support for implementation of complete systems based in fuzzy logic and even allowing the composition of rules with fuzzy and non-fuzzy elements transparently. In the next section, some developed experiments to evaluate these modifications are presented.

5. Tests and results

Tests were performed using the implementation of paradigm in the form of a C++ framework defined in [9], after its adjusts to support fuzzy concepts. Comparisons were performed with a system developed in C++ without using NOP. The system consists of a simulated washing machine controller, that defines the wash time according to the amount of dirt and the size of stain in clothes. It's rules and variables were provided by a fuzzy system expert as an example, not representing a real system. The Fuzzy Rulebase relating the input variables and the output variable has nine *Rules* as shown in table 1.

	No Stain - NS	**Medium Stain - MS**	**Big Stain - BS**
Little Dirt - LD	Very Short - VS	Average - A	Long - L
Average Dirt - AD	Short - S	Average - A	Long - L
Much Dirt - MD	Average - A	Long - L	Very Long - VL

Table 1. Block diagram of a RE-FAM.

Figure 5 shows the fuzzy sets for each defined linguistic variable.

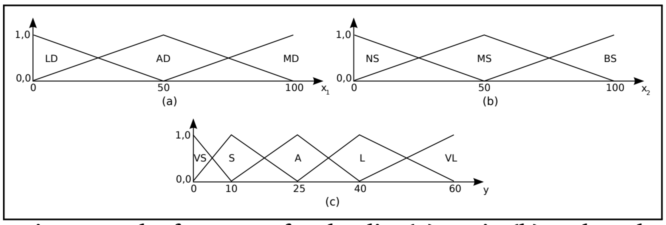


Figure 5. The fuzzy sets for the dirt (a), stain (b) and wash time (c) variables.

In the developed fuzzy system using the NOP framework, extended to support the fuzzy concepts, the *Methods* executed by the activated *Rules* also increments the value of a global variable that indicates how many *Rules* where activated during the program execution. For "defuzzyfication" it was utilized the centroid function over the resulting fuzzy set and the function granularity was adjusted to 0.1 in both systems.

The test case consists in assigning artificially generated values and comparing the results of both systems. The following code was used in the tests:

```
for i = 0 to 100 inc 10
        for j = 0 to 100 inc 10
                dirt = i
                stain = j

                perform inference

                write execution time in file
        end
end
```

This program resulted in the execution of the inference process 121 times. In both systems (fuzzy C++ and fuzzy NOP), measurements of the execution time between the assignment of the first value and the completion of inference were performed. For the tests, it was considered the total execution time of all inferences, that is, the sum of all measured times.

During the tests both systems calculated the same values for the passed input data set. Furthermore, an analysis of the amount of the *Rules* activated was performed. As the inference process was executed 121 times and the C++ fuzzy system analyzed the nine rules this amount of times, this resulted in 1089 verifications. However, in the developed NOP system, the execution of this test resulted in activation of only 277 *Rules*.

Despite this result, the C++ system average execution time was 0.366214 seconds while the NOP system developed with the modified framework was 1.12508 seconds, resulting in an execution ratio of 3.072:1. This was caused by the framework overhead since it was developed to simulate the NOP parallelism in a sequential execution environment by processing one layer of NOP entities (*Attributes*, *Premises*, ...) at each time. Furthermore, the framework was not optimized to use multiple threads.

6. Conclusion

With this work it was possible to verify the possibility of adapting the notification oriented paradigm to support the development of fuzzy logic systems. The changes mentioned in this work allowed the paradigm to be used for an proof-of-concept fuzzy system development. Furthermore, those changes allow the development of mixed systems, where crisp *Premises* and fuzzy *Premises* can compose the same *Rule* through the use of the logical connectives defined by Mamdani, and this can be seen as a contribution of this paper.

During the tests it was verified that there was a reduction in the amount of evaluated rules, which could result in reduction of necessary computation to execute the mentioned test case. This is due the inference mechanism of the paradigm, where the processing will occur only if there's a change of state in the system. However, it was also verified that, although the reduced amount of verified rules, the execution time of the system developed with the fuzzy NOP framework was three times bigger than the system developed directly in C++. This happened because the framework was developed to simulate the NOP parallelism in a sequential execution environment.

The next step of this work is to extend the NOP language [14], which is already under development, to include the proposed fuzzy concepts and structures. This new language will allow developers to easily create fuzzy systems, which will help in creation of new test cases and in improvement of the fuzzy NOP framework, in order to reduce its overhead.

References

[1] L. A. Zadeh: Fuzzy Sets, Information and Control, vol. 8, pp. 338-352. (1965).

[2] E.H. Mamdani: Application of fuzzy algorithms for control of simple dynamic plant, Proceedings - Institution of Electrical Engineers, vol. 121, no. 12, pp. 1585-1588. (1974).

[3] R. F. Banaszewski: Notification Oriented Paradigm - Advances and Comparisons (in portuguese), Curso de

Pós-Graduação em Engenharia Elétrica (CPGEI), Universidade Tecnológica Federal do Paraná (UTFPR), Curitiba, PR. (2009).

[4] E. Peters, R. P. Jasinsk, V. A. Pedroni and Jean M. Simão: A new hardware coprocessor for accelerating Notification-Oriented applications, Proceedings - International Conference on Field-Programmable Technology, pp. 257-260. (2012).

[5] S. Sánchez-Solano, A. J. Cabrera, I. Baturone, F. J. Moreno-Velo and M. Brox: FPGA implementation of embedded fuzzy controllers for robotic applications, IEEE Transactions on Industrial Electronics, vol. 54, no. 4, pp. 1937-1945. (2007. publisher=IEEE

[6] N. Sulaiman, Z. A. Obaid, M. H. Marhaban and M. N. Hamidon: FPGA-Based Fuzzy Logic : Design and Applications - a Review, IACSIT International Journal of Engineering and Technology, vol. 1, no. 5, pp. 491-503. (2009).

[7] J. M. Simão: A Contribution to the Development of a HMS Simulation Tool and Proposition of a Meta-Model for Holonic Control, Curso de Pos-Graduasao em Engenharia Eletrica (CPGEI), Universidade Tecnologica Federal do Parana (UTFPR), Curitiba, PR. (2005).

[8] J. M. Simão and P. C. Stadzisz: Notification Oriented Paradigm (NOP) - A Composition Technique and Notification Oriented software execution (in portuguese), Patent pending submitted to INPI/Brazil in 2008 and UTFPR Innovation Agency. (2008).

[9] A. F. Ronszcka: Contribution to the design of applications in Notification Oriented Paradigm (NOP) under the perspective of patterns (in portuguese), Curso de Pós-Graduação em Engenharia Elétrica (CPGEI), Universidade Tecnológica Federal do Paraná (UTFPR), Curitiba, PR. (2012).

[10] J. M. Simão, C. A. Tacla, R. F. Banaszewski, P. C. Stadzisz: Notification Oriented Paradigm (NOP) and Imperative Paradigm: A Comparative Study, Journal of Software Engineering and Applications (JSEA). (2012).

[11] T. J. Ross: Fuzzy Logic with Engineering Applications, John Wiley & Sons. (2004).

[12] J. M. Simão, J. A. Fabro, P. C. Stadzisz, L. V. R. Arruda, S. Ishimatsu: An Agent-Oriented Fuzzy Inference Engine, VI Simposio Brasileiro de Automasao Inteligente, Bauru, SP, pp. 585-590. (2003).

[13] J. T. S. de Souza, J. A. Fabro, J. M. Simão, R. F. Banaszewski: Proposal of a fuzzy inference engine using Notification Oriented Paradigm (NOP) (in portuguese), XIV Seminário de Iniciação Científica e Tecnológica da UTFPR, Pato Branco, PR. (2009).

[14] R. D. Xavier: Software development paradigms: comparing event-driven and notification oriented approaches (in portuguese), Curso de Pós-Graduação em Computação Aplicada (PPGCA), Universidade Tecnológica Federal do Paraná (UTFPR), Curitiba, PR. (2014).

[15] W. Pedrycz, F. Gomide: An Introduction to Fuzzy Sets: Analysis and Design, MIT Press. (1998).

[16] J. M. Simao, P. C. Stadzisz: Inference Based on Notifications: A Holonic Metamodel Applied to Control Issues, IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans, vol. 39, no. 1, pp. 238-250. (2009).

[17] J. M. Simão, P. C. Stadzisz: Conflict resolution mechanism and determinism warranty for Notification Oriented Paradigm (NOP) (in portuguese), Patent pending submitted to INPI/Brazil in 2010 and UTFPR Innovation Agency. (2010).

[18] J. M. Simão, P. C. Stadzisz, R. F. Banaszewski, C. A. Tacla: Optimized inference engine for Notification Oriented Paradigm (NOP) and mechanisms for conflict resolution to uniprocessor and multiprocessor environments applied to NOP (in portuguese), Patent pending submitted to INPI/Brazil in 2010 and UTFPR Innovation Agency. (2010).

SCIENTIFIC REPORT

f-correlated fuzzy numbers applied to HIV model with protease inhibitor therapy

V. M. Cabral[1], R. A. C. Prata[1], L. C. Barros[2*]
[1]Department of Mathematics, Federal University of Amazonas, Manaus, AM, Brazil
valtemircabral@yahoo.com.br
praroberto@gmail.com
[2]Department of Applied Mathematics, IMECC, University of Campinas, Campinas, SP, Brazil
laeciocb@ime.unicamp.br

Abstract. This paper presents the concept of f-correlated fuzzy numbers, which generalizes the concept of completely correlated numbers fuzzy, whose correlation is established from a straight line. In our case, the formulation is made with the aid of an injective monotone function. We include the case in which the correlation function is hyperbolic and get to addition and multiplication operations between correlated hyperbolic fuzzy numbers. Some properties of correlated fuzzy numbers are presented. Finally, we study a model with delay for the evolution of HIV in which the delay and the mortality rate of the virus, due to the action of the drug, are hyperbolically correlated parameters.

Keywords. f-correlated fuzzy numbers; extension principle; joint possibility distribution; hyperbolically fuzzy numbers; HIV dynamics

1. Introduction

In this paper we study a new form of interactivity between two fuzzy numbers ([4], [5]) and some of their properties. This interactivity is achieved through a injective monotone function, which we call f-correlated. The concept of fuzzy numbers correlated this concept generalizes the notion of fuzzy numbers correlated completely fuzzy numbers correlated ([7], [6]).

We present the operations of addition and product between two f-correlated fuzzy numbers and we emphasize those cases where the function is a straight line (in which case the fuzzy numbers are called correlated completely fuzzy numbers and that the function is hyperbolic, in the case, we call correlated hyperbolically fuzzy numbers and besides we study some properties of f-correlated fuzzy numbers correlated for example, study the calculation of the central value of an integrable function $g:\mathbb{R}^2\to\mathbb{R}$ and also the measure of interaction between two f-correlated fuzzy numbers correlated.

Finally, we present a model of HIV in which the delay (between the infection of cell by the HIV virus and the production of a new cell) and the mortality rate of the virus, are hyperbolically correlated fuzzy parameters. The fuzzy solution of the model is obtained by applying the extension principle for f-correlated fuzzy numbers in solving deterministic model of HIV.

2. Basic concepts

Definition 1 *Let $f:\mathbb{R}^n\longrightarrow\mathbb{R}^k$ be a function, $A_1,\dots,A_n$ interactive fuzzy numbers with joint possibility distribution C. The extension of f applied the $(A_1,\dots,A_n)$ via C is the fuzzy subset $f_C(A_1,\dots,A_n)$ of $\mathbb{R}^k$ which the membership function is given by*

$$\mu_{f_C(A_1,\dots,A_n)}(y)=\begin{cases}\sup_{(x_1,\dots,x_n)\in f^{-1}(y)}\mu_C(\mu_{A_1}(x_1),\dots,\mu_{A_n}(x_n)) & \text{if } f^{-1}(y)\neq\emptyset\\ 0 & \text{if } f^{-1}(y)=\emptyset.\end{cases}$$

where $f^{-1}(y)=\{(x_1,\dots,x_n)\in\mathbb{R}^n:(x_1,\dots,x_n)=y\}$.

Regarding the extension principle we have:

Proposition 1 *Let $A_1,\dots,A_n$ be fuzzy numbers, C a joint possibility distribution with marginal distributions $A_1,\dots,A_n$ and $f:\mathbb{R}^n\longrightarrow\mathbb{R}$ a continuous function. Then,*

$$[f_C(A_1,\dots,A_n)]^\alpha=f([C]^\alpha),$$

for all $\alpha\in[0,1]$. Furthermore, $f_C(A_1,A_2,\dots,A_n)$ is always a fuzzy number.

The proof of this result can be found in ([7],[3]).

3. f-correlated fuzzy numbers

Definition 2 *Let $f:X\longrightarrow Y$, $X,Y\in\mathbb{R}$ be a injective monotone function and $A,B\in\mathscr{E}(\mathbb{R})$ fuzzy numbers. We say that A and B are correlated via function or f-correlated, if its joint possibility distribution C is given by*

$$\mu_C(x,y)=\mu_A(x)\mathscr{X}_{\{y=f(x)\}}(x,y)=\mu_B(y)\mathscr{X}_{\{y=f(x)\}}(x,y)\tag{14}$$

where,

$$\mathscr{X}_{\{y=f(x)\}}(x,y)=\begin{cases}1 & \text{if} \quad y=f(x)\\ 0 & \text{if} \quad y\neq f(x)\end{cases}$$

It is interesting to note that from (14) the unique elements $(x,y)\in\mathbb{R}^2$ that have nonzero pertinence in C are those on the curve $\gamma(x)=(x,f(x))$.

In this case we have:

$$\begin{aligned}[B]^\alpha &= \{y \in \mathbb{R} : \mu_B(y) \geq \alpha\}\\ &= \{f(x) \in \mathbb{R} : \mu_A(x) \geq \alpha\}\\ &= f(\{x \in \mathbb{R} : \mu_A(x) \geq \alpha\})\\ &= f([A]^\alpha), \forall \alpha \in [0,1]. \qquad (15)\end{aligned}$$

$\mu_B(y) = \mu_A(f^{-1}(y))$, $\forall y \in \mathbb{R}$ if $[A]^\alpha = [a_1^\alpha, a_2^\alpha]$,

$$\begin{aligned}[C]^\alpha &= \{(x,y) \in \mathbb{R}^2 : \mu_C(x,y) \geq \alpha\}\\ &= \{(x,y) \in \mathbb{R}^2 : \mu_A(x)\mathscr{X}_{\{y=f(x)\}}(x,y) \geq \alpha\}\\ &= \{(x,f(x)) : \mu_A(x) \geq \alpha\}\\ &= \{(x,f(x)) : x \in [a_1^\alpha, a_2^\alpha]\}. \qquad (16)\end{aligned}$$

4. Operations with f-correlated fuzzy numbers

Let C be a joint possibility distribution of the f-correlated fuzzy numbers A, B and $g,h : \mathbb{R}^2 \longrightarrow \mathbb{R}$ functions given by

$$g(x,y) = x + y \text{ and } h(x,y) = xy.$$

The operations of *addition* and *product* of two f-correlated fuzzy numbers A, B, via C, are denoted by

$$A +_C B = g_C(A,B) \text{ and } A._C B = h_C(A,B).$$

So

$$\mu_{(A+_C B)}(z) = \sup_{z=x+y} \mu_C(x,y) \text{ and } \mu_{(A._C B)}(z) = \sup_{z=xy} \mu_C(x,y).$$

Since g_C and h_C are continuous functions from 1

$$\begin{aligned}[A +_C B]^\alpha &= [g_C(A,B)]^\alpha\\ &= \overline{\{z \in \mathbb{R} : \sup_{z=x+y} \mu_C(x,y) > \alpha\}}\\ &= \overline{\{z \in \mathbb{R} : \sup_{z=x+y} \mu_A(x)\mathscr{X}_{\{y=f(x)\}}(x,y) > \alpha\}}\\ &= \overline{\{x + f(x) : \mu_A(x) > \alpha\}}. \qquad (17)\end{aligned}$$

and

$$\begin{aligned}[A._C B]^\alpha &= [h_C(A,B)]^\alpha\\ &= \overline{\{z \in \mathbb{R} : \sup_{z=xy} \mu_C(x,y) > \alpha\}}\\ &= \overline{\{z \in \mathbb{R} : \sup_{z=xy} \mu_A(x)\mathscr{X}_{\{y=f(x)\}}(x,y) > \alpha\}}\\ &= \overline{\{xf(x) : \mu_A(x) > \alpha\}}. \qquad (18)\end{aligned}$$

Example 1 *When $f(x) = qx + r$, $q \neq 0$, the fuzzy numbers A and B are called completely correlated ([7], [3]). In this context, the possibility of joint distribution of the A and B is given by*

$$\mu_C(x,y) = \mu_A(x)\mathscr{X}_{\{qx+r=y\}}(x,y) = \mu_B(y)\mathscr{X}_{\{qx+r=y\}}(x,y) \qquad (19)$$

where,

$$\mathscr{X}_{\{qx+r=y\}}(x,y) = \begin{cases} 1 & \text{if} \quad qx + r = y\\ 0 & \text{if} \quad qx + r \neq y\end{cases}$$

is the characteristic function of the straight $\{(x,y) \in \mathbb{R}^2 : y = qx + r\}$.

In this case, we have:

$[A]^\alpha = [a_1^\alpha, a_2^\alpha]$;

$[C]^\alpha = \{(x, qx + r) \in \mathbb{R}^2 : x = (1-s)a_1^\alpha + sa_2^\alpha, s \in [0,1]\}$;

$[B]^\alpha = q[A]^\alpha + r$, *for all* $\alpha \in [0,1]$ *and* $\mu_B(x) = \mu_A(\frac{x-r}{q})$, $\forall x \in \mathbb{R}$.

According to formula 17 the α-levels of $A +_C B$ are given by

$$\begin{aligned}[A +_C B]^\alpha &= \overline{\{x + y \in \mathbb{R} : \mu_A(x) > \alpha, qx + r = y\}}\\ &= \overline{\{(q+1)x + r \in \mathbb{R} : \mu_A(x) > \alpha\}}\\ &= (q+1)\overline{\{x \in \mathbb{R} : \mu_A(x) > \alpha\}} + r\\ &= (q+1)[A]^\alpha + r.\end{aligned}$$

Note that, when $q = -1$ the sum of the completely correlated fuzzy numbers A and B is real number r. Furthermore, if $r = 0$, $A +_C B = 0$ ([7]).

Example 2 *When $f(x) = \frac{q}{x} + r$, with $x > 0$ and $q \neq 0$, the fuzzy numbers A and B, with $0 \notin [A]^0$, are called correlated hyperbolically. In this context, the joint possibility distribution C of A and B has the following membership function*

$$\mu_C(x,y) = \mu_A(x)\mathscr{X}_{\{\frac{q}{x}+r=y\}}(x,y) = \mu_B(y)\mathscr{X}_{\{\frac{q}{x}+r=y\}}(x,y) \qquad (20)$$

where,

$$\mathscr{X}_{\{\frac{q}{x}+r=y\}}(x,y) = \begin{cases} 1 & \text{if} \quad \frac{q}{x} + r = y\\ 0 & \text{if} \quad \frac{q}{x} + r \neq y\end{cases}$$

is the characteristic function of the hyperbole $\{(x,y) \in \mathbb{R}^2 : \frac{q}{x} + r = y\}$.

In this case if $[A]^\alpha = [a_1^\alpha, a_2^\alpha]$, we have:

$[C]^\alpha = \{(x, \frac{q}{x} + r) \in \mathbb{R}^2 : x \in [a_1^\alpha, a_2^\alpha]\}$;

$[B]^\alpha = \frac{q}{[A]^\alpha} + r = [\frac{q}{a_2^\alpha} + r, \frac{q}{a_1^\alpha} + r]$;

$\mu_B(x) = \mu_A(\frac{q}{x-r})$.

The α-levels of $A._C B$ are give by

$$\begin{aligned}[A._C B]^\alpha &= \{(x,y) \in \mathbb{R} : \mu_{h_C(A,B)}(x,y) \geq \alpha\}\\ &= \{(x,y) \in \mathbb{R} : \sup_{z=xy} \mu_C(x,y) \geq \alpha\}\\ &= \{(x,y) \in \mathbb{R} : \sup_{z=xy} \mu_A(x)\mathscr{X}_{\{\frac{q}{x}+r=y\}}(x,y) \geq \alpha\}\\ &= \overline{\{xy \in \mathbb{R} : \mu_A(x) > \alpha, \frac{q}{x} + r = y\}}\\ &= \overline{\{x(\frac{q}{x} + r) \in \mathbb{R} : \mu_A(x) > \alpha\}}\\ &= \overline{\{q + xr \in \mathbb{R} : \mu_A(x) > \alpha\}}\\ &= q + r[A]^\alpha.\end{aligned}$$

Note that for $r = 0$ the product of the correlated hyperbolically fuzzy numbers A and B is the real number real q. Furthermore, when $q = 1$, $A._c B = 1$.

5. Properties of the f-correlated fuzzy numbers

In this section we present some properties of the f-correlated fuzzy numbers, such as the calculation of the central value of a function integrable $g : \mathbb{R}^2 \to \mathbb{R}$ and the measure of interactivity between two f-correlated fuzzy numbers.

Definition 3 *Let C be a joint possibility distribution in $\mathbb{R}^n$, with $\alpha \in [0,1]$ and let be $g : \mathbb{R}^n \to \mathbb{R}$ function integrable. The central value of g in $[C]^\alpha$ is given by*

$$\begin{aligned}\mathscr{C}_{[C]^\alpha}(g) &= \frac{1}{\int_{[C]^\alpha} dx} \int_{[C]^\alpha} g(x)dx \\ &= \frac{1}{\int_{[C]^\alpha} dx_1...dx_n} \int_{[C]^\alpha} g(x_1,..,x_n)dx_1...dx_n \quad (21)\end{aligned}$$

For the case in which $[C]^\alpha$ is degenerate for same $\alpha \in [0,1]$, the central value of g is given by

$$\mathscr{C}_{[C]^\alpha}(g(x)) = \lim_{\epsilon \to 0} \frac{1}{\int_{S(\epsilon)} dx} \int_{S(\epsilon)} g(x)dx. \quad (22)$$

where,

$$S(\epsilon) = \{x \in \mathbb{R}^2 : \exists c \in [C]^\alpha, ||x - c|| \le \epsilon\}$$

Definition 4 *Let $A, B : \mathbb{R} \to [0,1]$ be f-correlated fuzzy numbers and $g : \mathbb{R}^2 \to \mathbb{R}$ a integrable function and $C : \mathbb{R}^2 \to [0,1]$ the joint possibility distribution of A and B. The central value of g is*

$$\mathscr{C}_{[C]^\alpha}(g) = \lim_{\epsilon \to 0} \frac{1}{\int\int_{S(\epsilon)} dxdy} \int\int_{S(\epsilon)} g(x,y)dxdy \quad (23)$$

where,

$$[C]^\alpha = \{(x, f(x)\}; x \in [a_1^\alpha, a_2^\alpha]\} \quad and$$

$$S(\epsilon) = \{x \in \mathbb{R}^2 : \exists c \in [C]^\alpha \quad with \quad ||x - c|| \le \epsilon\}.$$

Example 3 *For the projections π_x, π_y and for $f(x) = qx + r$ and according to [8] we have,*

$\mathscr{C}_{[C]^\alpha}(\pi_x) = \frac{a_1^\alpha + a_2^\alpha}{2}$,

$\mathscr{C}_{[C]^\alpha}(\pi_y) = q\frac{a_1^\alpha + a_2^\alpha}{2} + r$ *and*

$\mathscr{C}_{[C]^\alpha}(\pi_x\pi_y) = q\frac{(a_2 - a_1)^2}{3} + \frac{2qa_1a_2 + (a_1 + a_2)r}{2}$.

Example 4 *For the projections π_x, π_y and for $f(x) = \frac{q}{x} + r$ we have*

$\mathscr{C}_{[C]^\alpha}(\pi_x) = \frac{a_1^\alpha + a_2^\alpha}{2}$ *and*

$\mathscr{C}_{[C]^\alpha}(\pi_y) = \frac{b_1^\alpha + b_2^\alpha}{2} = q(\frac{\frac{1}{a_2^\alpha} + \frac{1}{a_1^\alpha}}{2}) + r.$

For α fixed, we have:

$$[C]^\alpha = \{(\frac{q}{y - r}, y); y \in [b_1^\alpha, b_2^\alpha]\} = \{(c(y), y); y \in [b_1^\alpha, b_2^\alpha]\}.$$

Finally,

$$\begin{aligned}\mathscr{C}_{[C]^\alpha}(\pi_x\pi_y) &= \frac{1}{b_2^\alpha - b_1^\alpha}\int_{b_1}^{b_2} yc(y)dy \\ &= \frac{1}{b_2^\alpha - b_1^\alpha}\int_{b_1}^{b_2} \frac{yq}{y - r}dy \\ &= \frac{1}{b_2^\alpha - b_1^\alpha}\int_{b_1}^{b_2} (q - \frac{qr}{y - r})dy \\ &= \frac{1}{b_2^\alpha - b_1^\alpha}[q(b_2 - b_1) - qr\ln(\frac{b_2^\alpha - r}{b_1^\alpha - r})] \\ &= q + \frac{qr}{b_2^\alpha - b_1^\alpha}\ln(\frac{b_2^\alpha - r}{b_1^\alpha - r})\end{aligned}$$

Definition 5 *Let A and B be fuzzy numbers and C the joint possibility distribution of A and B. The measure of interactivity between the α-levels of the fuzzy subsets A and B is defined by*

$$\mathscr{R}_{[C]^\alpha}(\pi_x\pi_y) = \mathscr{C}_{[C]^\alpha}(\pi_x\pi_y) - \mathscr{C}_{[C]^\alpha}(\pi_y)\mathscr{C}_{[C]^\alpha}(\pi_x). \quad (24)$$

Example 5 *For the completely correlated fuzzy numbers*

$$\begin{aligned}\mathscr{R}_{[C]^\alpha}(\pi_x\pi_y) &= \mathscr{C}_{[C]^\alpha}(\pi_x\pi_y) - \mathscr{C}_{[C]^\alpha}(\pi_y)\mathscr{C}_{[C]^\alpha}(\pi_x) \\ &= \frac{q(a_2^\alpha - a_1^\alpha)^2}{12}.\end{aligned}$$

Example 6 *When the function that define the interactivity between the fuzzy numbers A and B is the hyperbole $f(x) = \frac{q}{x} + r$, the measure of the interactivity is given by*

$$\begin{aligned}\mathscr{R}_{[C]^\alpha}(\pi_x\pi_y) &= \mathscr{C}_{[C]^\alpha}(\pi_x\pi_y) - \mathscr{C}_{[C]^\alpha}(\pi_y)\mathscr{C}_{[C]^\alpha}(\pi_x) \\ &= -\frac{q(a_2^\alpha - a_1^\alpha)^2}{4a_1^\alpha a_2^\alpha} + r[\frac{a_1^\alpha a_2^\alpha}{a_1^\alpha + a_2^\alpha}\ln(\frac{a_1^\alpha}{a_2^\alpha}) - \frac{a_1^\alpha + a_2^\alpha}{2}].\end{aligned}$$

6. HIV dynamic with delay and death rate of virus correlated hyperbolically

Let us start considering the following model for the dynamics of HIV.

$$\begin{cases} \frac{dx(t)}{dt} = \lambda - cx(t) - \beta(t)x(t)v(t) \\ \frac{dy(t)}{dt} = \beta(t)x(t)v(t) - ay(t) \\ \frac{dv(t)}{dt} = k(t)y(t) - uv(t), \end{cases} \quad (25)$$

where,

- $x(t)$ is the population of the cell uninfected;
- $y(t)$ is the population of the cell infected that produce virus;
- $v(t)$ is the population of the virus;
- λ is the influx blood;
- c is the death rate of the cells uninfected;
- $\beta(t)x(t)v(t)$ is the rate of the produce of the cells infected;
- u is the rate of the decline of the virus concentration;
- a is the death rate of the cells infected;
- $k(t)$ is the rate of the produce free virus particles;

This model has been used to quantify the dynamics of the virus in individuals who are in antiretroviral therapy ([11], [12], [13],[2]). In our study it is necessary to consider the delay between the intracellular infection of a new cell and the production of new virus particle [9].

The delay τ is the time between the infection of a cell by a virus and the production of a new particle of virus. This implies that the recruitment of virus-producing cells, at time t, is given by the density of cells that were newly infected at time $t-\tau$ and are still alive at time t.

When a cell is infected, it takes a time τ_{int} to that are produced particle of virus and after application of any anti-viral drug there is a delay in pharmacological effect due to the time needed for the absorption, distribution and penetration into the target cells. We will assume that

$$\tau = \tau_{int} + \tau_{farm}.$$

Incorporating the delay τ, the system (25) shall be given by the following system of differential equations

$$\frac{dx(t)}{dt} = \lambda - cx(t) - \beta(t)x(t)v(t) \quad (26)$$

$$\frac{dy(t)}{dt} = \beta(t-\tau)x(t-\tau)v(t-\tau)e^{-\widetilde{a}\tau} - ay(t) \quad (27)$$

$$\frac{dv(t)}{dt} = k(t)y(t) - uv(t), \quad (28)$$

where,

- $\widetilde{a}$ is the death rate of the cells uninfected, but that does not produce virus;
- $e^{-\widetilde{a}\tau}$ is the probability of survival of infected cells of the time $t-\tau$ to the time t. Generally, the probability of survival is given by function $f(\tau)$ with $0 \le f(\tau) \le 1$.

The equation (27) has delay and the analytical solutions are difficult of be obtained. However, for this specific case the population of cells uninfected; infected virus-producing cells and virus free are in a steady-state level before treatment [10]. These facts facilitate mathematical analysis and allow that a simple analytic solutions are deduced. The nontrivial solution for the steady state is given by

$$x_0 = \frac{au}{\beta k}e^{\widetilde{a}\tau} \quad (29)$$

$$y_0 = \frac{\lambda}{a}e^{\widetilde{a}\tau} - \frac{uc}{\beta k} \quad (30)$$

$$v_0 = \frac{ky_0}{u}, \quad (31)$$

where β and k are constant of pretreatment rates.

6.1 Protease Inhibitor Therapy In the subsection we will apply the concept of hyperbolically correlated fuzzy numbers in a HIV model with treatment in which the death rate of virus u and the time of action of the pharmaco τ are correlated. According Herz [10] these parameters are statistically correlated and moreover, they are related by the equation $\tau = 1 - \frac{1}{u}$ this justifies we consider τ and u hyperbolically correlated fuzzy numbers. In [9] there exist a model similar to this, but u and τ are models by noninteractive fuzzy numbers, namely, the joint possibility distribution is given by minimum t-norm.

In the work of [9] and [3] are included the treatment of HIV by the use of protease inhibitors. This type treatment block the production of new infectious virus v_I from already infected cells, allowing only noninfectious virus to be generated. The infectious virus declines, but the cells continues to infect [1] and [14].

Second [10] the equation (28) also describes the total dynamics of free virus. However, the infectious virus are not produced in time $t > 0$ and decline according to the equation

$$\frac{dv_I(t)}{dt} = -uv_I(t), \quad (32)$$

and the equations (26)and (27) remain valid.

According to [10], in scale of time considered, the population of uninfected cells remains constant $x(t) = x_0$.

When $x(t) = x_0$ and $v_I(t)$ decline exponentially, we have

$$y(t) = \frac{y_0}{a-u}[ae^{-u(t-\tau)} - ue^{-a(t-\tau)}] \;\; for \;\; t > \tau. (33)$$

From equation (28) the time of evolution of the free virus is

$v(t) = v_0 \;\; para \;\; 0 < t \le \tau$ and

$$v(t) = v_0e^{-u(t-\tau)} + \frac{uv_0}{a-u}\left(\frac{u}{a-u}[e^{-a(t-\tau)} - e^{-u(t-\tau)}]\right) + \frac{uv_0}{a-u}[a(t-\tau)e^{-u(t-\tau)}] \;\; para \;\; t > \tau. \quad (34)$$

6.2 Fuzzy solution for free virus population with delay and virus death rate hyperbolically correlated Second [10], the delay (τ), virus death rate (u) and death rate of the infected cells (a) are related by

$$T = \tau + \frac{1}{u} + \frac{1}{a},$$

where T is the medium time of viral generation.

According [10], the medium time of viral generation varies between $T = 2.5$ day and $T = 3.1$ days.

We will adopt $T = 3$ days and we will assume that $a = 0.5/day$.

Suppose that τ and u are hyperbolically correlated by

$$\tau = 1 - \frac{1}{u}.$$

According to [9] the delay τ can be given by fuzzy parameter, and it can be modeled by triangular fuzzy number

$$\Gamma = (0.08; 0.53; 0.98) \text{ or}$$
$$[\Gamma]^\alpha = [0.08 + 0.45\alpha, 0.98 - 0.45\alpha].$$

The virus death rate u is modelled by the fuzzy numbers U such that

$$[U]^\alpha = \frac{1}{1 - [\Gamma]^\alpha} = [\frac{1}{0.92 - 0.45\alpha}, \frac{1}{0.02 + 0.45\alpha}].$$

The joint possibility distribution C of Γ and U has the following membership function

$$\mu_{_C}(u, \tau) = \mu_{_\Gamma}(\tau)\mathscr{X}_{\{1 - \frac{1}{u} = \tau\}}(u, \tau),$$

and we have

$$\begin{aligned}[C]^\alpha \quad = \quad & \{(u, 1 - \frac{1}{u}) : u = \\ & (1 - s)[\frac{1}{0.92 - 0.45\alpha}] + s[\frac{1}{0.02 + 0.45\alpha}], \\ & s \in [0, 1]\}.\end{aligned}$$

The central value of the equation (34) is a curve in t, the which is obtained by

$$\begin{aligned}&\mathscr{C}_{[C]^\alpha}(v_t(u,\tau)) = \frac{1}{\int_{[C]^\alpha} dud\tau} \int_{[C]^\alpha} v_t(u,\tau) dud\tau = \\ &\frac{1}{u_2^\alpha - u_1^\alpha} \int_{u_1^\alpha}^{u_2^\alpha} v_t(u,\tau(u)) dud\tau = \\ &\frac{1}{u_2^\alpha - u_1^\alpha} \int_{u_1^\alpha}^{u_2^\alpha} v_0 e^{-u(t-1+\frac{1}{u})} + \frac{uv_0}{a-u}(\frac{u}{a-u}[e^{-a(t-1+\frac{1}{u})} - \\ &e^{-u(t-1+\frac{1}{u})}]) + \\ &\frac{uv_0}{a-u}[a(t-1+\frac{1}{u})e^{-u(t-1+\frac{1}{u})}] \qquad (35)\end{aligned}$$

A solution to (35) can be approximated by any numerical existing in the literature. The solution graph for the α-level= 0.3 and for $a = 0.4, \tilde{a} = 0.5, \lambda = 0.8, \beta = 0.2, c = 0.3.u = 0.5, k = 0.3$ are illustrated below

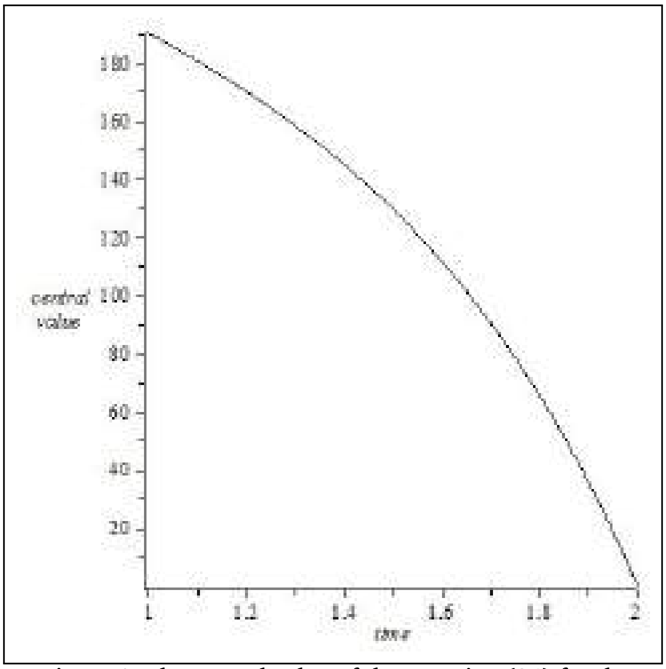


Figure 1. The central value of the equation (34) for the α-level= 0.3 and for $a = 0.4, \tilde{a} = 0.5, \lambda = 0.8, \beta = 0.2, c = 0.3.u = 0.5, k = 0.3$.

The fuzzy solution for viral load is obtained by the application of the Proposition 1 in the equation(34).

Therefore, the α-levels of the fuzzy solution for $t > \tau$ are given by

$$\begin{aligned}&[(v_t)_C(U,\Gamma)]^\alpha = (v_t)([C]^\alpha) = \\ &\{v_t(u,\tau) : (u,\tau) \in [C]^\alpha\} = \\ &\Big\{v_0 e^{-u(t-\tau)} + \frac{uv_0}{0.5-u}\Big(\frac{u}{0.5-u}\Big[e^{-0.5(t-\tau)} - e^{-u(t-\tau)}\Big]\Big) \\ &+ \frac{0.5uv_0}{0.5-u}(t-\tau)e^{-u(t-\tau)} : \quad \tau = 1 - \frac{1}{u} \text{ e } u \in [U]^\alpha\Big\} = \\ &\Big\{v_0 e^{-u(t+\frac{1}{u}-1)} + \frac{uv_0}{0.5-u}\Big(\frac{u}{0.5-u}\Big[e^{-0.5(t+\frac{1}{u}-1)} \\ &-e^{-u(t+\frac{1}{u}-1)}\Big]\Big) + \frac{0.5uv_0}{0.5-u}(t + \frac{1}{u} - 1)e^{-u(t+\frac{1}{u}-1)} \\ &: u \in [U]^\alpha\Big\}.\end{aligned}$$

An important remark is that the deterministic solution v_t for population free virus is obtained in the equation 34 is always contained in the solution fuzzy $(v_t)_C(\Gamma, U)$ for population free virus, namely, for each fixed τ , $\tau \in [\Gamma]^\alpha$ choice u such that $\tau = 1 - \frac{1}{u}$, thence

$$v_t \in [(v_t)_C(\Gamma, U)]^\alpha.$$

6.3 Conclusions In the this work we present a form of correlation between fuzzy numbers, which we call the f-correlated, because is obtained with the support of a injective monotonic function f. With this new form of interactivity between fuzzy numbers, we obtain the addition and product of two fuzzy numbers.

The operations are illustrated with the case where the function f is linear, hyperbolic and parabolic. In the case where f is linear a addition between two fuzzy numbers can generate a real number, for the case in which f is a hyperbolic verified that the product of two fuzzy numbers f-correlated can generate a real number and for fuzzy numbers parabolically correlated we have of possibility of obtaining a real number with the division of two f-correlated fuzzy numbers.

Finally, we study some of the properties of the fuzzy numbers f-correlated, such as, the calculation of central value and the measure of interactivity of two f-correlated fuzzy numbers.

Acknowledgments.The author third would like to thank CNPq (Brazil) for providing the financial support needed in this research (grant no. 305862/2013-8).

References

[1] A.Perelson, A. Neummann, M Markowitz, J. Leonard and D. Ho. HIV-1 dynamics in vivo: virion clearance rate, infected cell life-span and viral generation time. Science 271, 1582-186, 1996.

[2] X. Wei, S. K. Ghosh, M.E. Taylorz, V. A. Johnson, E. A. Emini, P. Deutsch, J. D. Lifson, S. Bonhoeffer, M. A. Nowak, B. H. Hahn, M. S. Saag and G. M. Shaw. Viral dynamics in human immunodeficiency virus type 1 infection. Nature 373, 117-122, 1995.

[3] V. M. Cabral. Equações diferenciais fuzzy com parâmetros interativos. IMECC–UNICAMP, Campinas/SP, Brazil (*in Portuguese*), 2011.

[4] D. Dubois and H. Prade. Additions of Interactive Fuzzy Numbers. IEEE Transaction on Automatic Control 26, 926-936, 1981.

[5] D. Dubois and H. Prade. Possibility Theory: An Approach to Computerized Processing of Uncertainty. Plenum Press, New York, 1988.

[6] C. Carlsson and R. Fullér. On possibilistic mean value and variance of fuzzy numbers. Fuzzy Sets and Systems 122, 315-369, 2001.

[7] C. Carlsson, R. Fúller and T. Keresztfalvi. Additions of completely correlated fuzzy numbers. Fuzzy IEEE 2004 CD-ROM Conference Proceedings, Budapest, 26-29, 2004.

[8] R. Fullér and P. Majlender. On interactive fuzzy numbers. Fuzzy Sets and Systems 143, 355-369, 2004.

[9] R. Jafelice, L. Barros and R. Bassanezi. A fuzzy delay differential equation model for HIV dynamics. Proceeding of IFSA-EUSFLAT Conference, Lisbon, Portugal, 2009.

[10] A. Herz, S. Bonhoeffer, R. Anderson, R. May and M. Nowak. Viray dynamics in vivo: Limitations on estimates of intracellular delay and virus decay. In. Proc. Natl. Acad. Sci (medical Sciences), 7247-7251, 1996.

[11] D. Ho, A. Neumann, A. Perelson, S Chen, J. Leonard and M. Markowitz. Rapid turnover of plasma virion and CD4 lymphocytes in HIV-1 infection. Nature (London) 373, 123-126, 1995.

[12] M. Nowak, S. Bonhoeffer, C. Loveday, P. Balfe, M. Semple, S. Kaye, M. Tenant-Flowers and R. Tedder. Nature (London) 375, 193-193, 1995.

[13] M. Nowak, S. Bonhoeffer, A. Hill, R. Boehme, H. Homas and H. McDade. Viral dynamics in hepatitis B virus infection. Proc. Natl. Acad. Sci (USA) 93, 4398-4402, 1996.

[14] D. Winslow and M. Otto. HIV protease inhitors. AIDS 9 (Suppl.A), S183-S192, 1995.

SCIENTIFIC REPORT

Synthesis of Fuzzy Pattern Trees by Cartesian Genetic Programming

Anderson R. dos Santos, Jorge Luís M. Amaral
State University of Rio de Janeiro, 524 São Francisco Xavier Street
andersonrds_rj@hotmail.com
jamaral@uerj.br

Abstract. This work presents a system for induction of fuzzy classifiers. Instead of the tra-ditional fuzzy based rules, it was used a model called Fuzzy Pattern Trees (FPT), which is a hierarchical tree-based model, having as internal nodes, fuzzy logical operators and the leaves are composed of a combination of fuzzy terms and the input attributes. The classifier was obtained by creating a tree for each class. This tree will be a "logic class description" which allows the interpretation of the re-sults. The learning method originally designed for generating a FPT was replaced by Cartesian Genetic Programming in order to provide a better exploration of the search space. The FPT classifier was compared against Support Vector Ma-chines, K Nearest Neighbors and Random Forests on several datasets from the UCI Machine Learning Repository and it presented competitive results. It was also compared with Fuzzy Pattern trees generated by the former learning method and presented comparable results with smaller trees.

Keywords. Machine Learning, Fuzzy Pattern Trees, Cartesian Genetic Programming, Classi-fication, interpretability.

1. Introduction

Recent technological advancements had allowed us to collect huge volume of data whose analysis in a reasonable time frame is beyond the human capacity. Therefore, it justifies the considerable interest in the study of models that can learn from a set of data [1]. The induction of these models can be done in an automatic fashion through the use of different approaches such as: Artificial Neural Networks, Bayesian methods, graphical models, decision trees, among others. However, when one wants to understand "how" the induced model makes its decision, that is, if you want to extract the knowledge of how the decision is made, then the models generated by symbolic approaches, such as, rules-based systems become more attractive, since they can generate interpretable models.

Some of the most successful methods applied in the synthesis of interpretable models are based on fuzzy set theory [2, 3], because fuzzy systems creates an interface between quantitative standards and qualitative knowledge structures expressed in natural language. This feature makes fuzzy systems attractive from the point of view of knowledge representation, allowing the acquired intelligence from a database to be represented in a comprehensible form. As a result, it gives the model a higher degree of interpretability [4].

This work describes a system for automatically induce models, applied in the classification tasks, based on fuzzy set theory. Instead of the traditional fuzzy based rules, it was used a model called Fuzzy Pattern Trees (FPT), which is a hierarchical tree-based model, having as internal nodes, fuzzy logical operators and the leaves are composed of a combination of fuzzy terms with the input attributes. In this paradigm, the classifier is obtained by creating a tree for each class, which will be a "logic class description" and as a consequence it allows the interpretation of the results.

Although, the current induction methods in FPT [5, 6] can produce competitive results, we propose to replace them by Cartesian Genetic Programming (CGP), on the grounds that CGP is a highly efficient and flexible form of Genetic Programming that encodes a graph representation of a computer program. Hence, they are used to represent several computational structures, and can be easily used to represent FPTs. Also, CGP is global search method capable to explore very large search spaces efficiently in order to find an appropriate representation of a FPT, which presents not only good accuracy but also interpretability.

This paper is divided into the following sections, first is presented a brief description of Fuzzy Patterns Trees. In the following section, a concise description of the Cartesian genetic programming is given. The fourth section describes the proposed method, the fifth discusses the obtained results and the last one states the conclusions.

2. Fuzzy Pattern Trees

The Fuzzy Pattern Trees (FPT) was created with the aim of representing the knowledge through an expression in the form of tree rather than represent it in the form of rules. The first induction method of FPT was created by Huang, Gedeon and Nikravesh [5], and later, the tree generation algorithm was refined in [6]. The adoption of this type of hierarchical representation seeks to minimize the problems existing in rule-based systems such as the exponential growth of a number of rules with the increase in number of entries and interpretability's commitment when a large amount of rules is generated to achieve accuracy requirements. Consequently, FPTs provides an alternative to build accurate and interpretable fuzzy models.

Fuzzy Pattern Trees are hierarchical models with a tree structure, in which the internal nodes are operators used in fuzzy systems and the leaves are composed of fuzzy terms associated with an input attribute. In the course of its evaluation, the FPT propagates the information from the bottom to the top. Therefore, the internal nodes receive values of their predecessors and combine them using an operator, showing the output at the top level. Hence, the FPTs execute a recursive mapping, generating the output in a unit interval. A classifier based on pattern tree is constructed by creating a tree for each class. In addition, the classification occurs in favor to the tree (class) with the higher output value. Also, since each tree can be considered as a "logic description" of a class, it allows a more specific interpretation of the learning problem [6].

A FPT example can be seen in Figure 1, where the tree represents the class "Good wine". The input attributes are the alcohol content, acidity, the concentration of Sulfur dioxide and sulfates. They are associated with a fuzzy term that represents a range in the attribute universe of discourse. For example in Figure 1, the fuzzy term Alcohol_Low, represents the fuzzy set that indicates low alcohol content. The membership value obtained in fuzzy sets is grouped by operators who maintain the partial results in the range $[0, 1]$. The value obtained in the output after all the groupings of features must get closer to 1, if the given attributes presented at the bottom of the tree represent the class well.

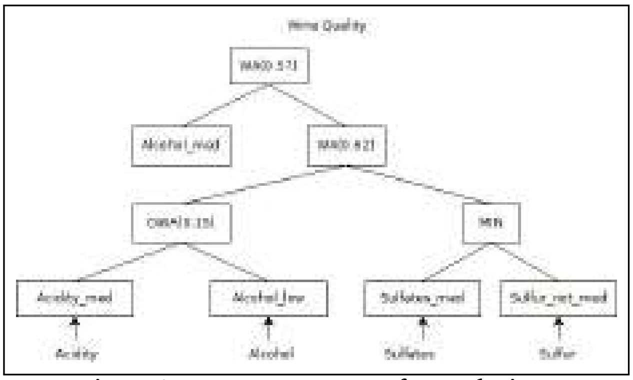


Figure 1. Fuzzy Pattern tree of a good wine.

The association between an attribute and a fuzzy term is represented by a membership function. The tree-based model maps various entries in only one output variable. The interpretation of the output produced can be seen as a model that simplifies the overall evaluation of a property, evaluating different criteria and aggregating these evaluations later [6]. If one looks again to Figure 1, it can be observed that the wine quality is related with two subtrees. The first relates the alcohol content and acidity while the other deals with the concentration of sulfate and sulfur dioxide. The information of these trees is then combined in a higher level. Subtrees represent different knowledge that must be combined. It is also relevant to note that the analysis of these subtrees can provide additional information. Depending on the operator in the top of the subtree, it is possible to identify which fuzzy terms contributes more with the final result. In addition to the fuzzy terms, t-norms, t-conorms and average operators like Ordered Weighted Average (OWA) and Weighted Average (WA) are also used in the tree creation [6].

The learning strategy proposed in [6] called "Beam Search" has some drawbacks: the first one is related to the "greedy" characteristic of the search algorithm, always looking for the best candidate in the current stage of construction. This feature can sometimes restrict the search space, making the algorithm trapped in a suboptimal point. Other disadvantage is associated with the "curse of dimensionality". If the amount of attributes and the width of the beam are large, the algorithm will take a long time to evaluate all the possibilities, for this reason, there will be an explosion in the amount of possible combinations. On the other hand, if the width of the beam is small, then only a small region of the search space will be explored. Consequently, it can hamper the algorithm task of finding good solutions; therefore in order to minimize these problems, we propose the substitution of the current learning method of FPT by CGP, since it is highly efficient global search method that can discover accu-rate and interpretable FPTs.

3. Cartesian Genetic Programming

Cartesian Genetic Programming (CGP) [7] is a form of genetic programming in which the programs are represented by graphs. The graph is encoded in a linear sequence of integers and is represented in a grid of computational nodes. Although, the grid can have any number of dimensions, in the majority of the applications, it has only one or two. Besides, it is worth noting that CGP presents several advantages such as: neutrality, redundancy and the absence of a problem called Bloat (program growth without significant return in terms of fitness), common in other methods of genetic programming [8, 9]. On CGP, programs are represented in a linear sequence of integers, which are called chromosomes or genotypes, as shown in Figure 2 (a). These chromosomes can be divided into parts, called genes that can be labeled in function, connection or output genes. A node is formed by the function gene and the connection genes. The function (labeled by the function gene) uses as input parameters the values indicated by the connection genes to generate a node output, which in turn can be used as input parameter of a function in another node. For the sake of development of the evolutionary process in the genotypes, it is necessary to decode them in phenotypes, to obtain the solution in the problem domain. When the genotype is decoded, some nodes may not be connected to the data stream, which creates an effect of neutrality. An example can be seen in Figure 2(b). While the genotypes have a fixed size, the phenotypes can vary in size. The mutation operator is the most important operator in CGP. It is commonly used the one-point mutation.

The amount of genes that suffer mutation operator is usually defined by a percentage of total genes and it is called mutation rate. Figures 2(a), 2(b), 2(c) and 2(d) show the before and after of a mutation operation. In this example, the last gene has changed; one can see that the change of this single gene can change significantly the phenotype [7]. The evolutionary algorithm used in CGP, is the evolutionary strategy $1 + \lambda$, where normally the value of λ is 4 [7]. This strategy allows CGP to find good solutions very efficiently in few evaluations. The next section describes how CGP can be used to synthesize Fuzzy Pattern Trees.

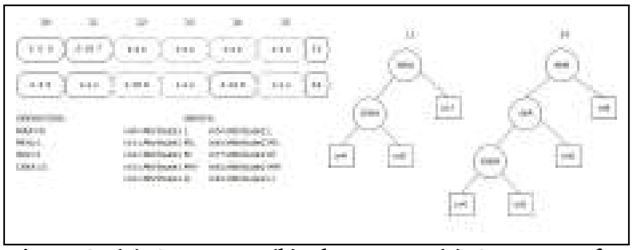


Figure 2. (a) Genotype. (b) Phenotype. (c) Genotype after

Mutation. (d) Phenotype after mutation.

4. Proposed Method: CGP-FPT

The proposed method uses the CGP for synthesis of FPTs. There are two main reasons to justify this choice: the first is related to the flexibility of the graph representation employed in the CGP, which can easily be used to represent a FPT; and the second is associated with the competence of CGP in exploiting very large search spaces efficiently, thus making the synthesis process less sensitive to the curse of dimensionality. Since the exploration of the search space on CGP does not rely on the greedy strategy of beam search, it has a better chance to achieve better solutions. Moreover, it is not restricted as the beam search (which is limited by the width of the beam). This also increases the chances of obtaining better solutions. As long as the proposed model utilizes an evolutionary algorithm, its success depends on how the solution is coded and on the choice of fitness function for evaluation of the solution. The next two subsections explain each of these implementations.

4.1 Representation Each attribute can be labeled by one of the five linguistic terms (fuzzy terms), referred to as *Low* (L), *Medium-Low* (ML), *Medium* (M), *Medium-high* (MH), *High* (H). The linguistic terms are obtained through the partitioning of the input attributes space. The function genes can represent the different operators that can be utilized in the FPT. In the interest of increasing the interpretability of the solution (tree), it was chosen to use a subset of the operators found in [6]. Thus, the following operators were employed: $Maximum = \max(a,b)$; $Minimum = \min(a,b)$; $WA(x) = xa + (1-x)b$; $OWA(x) = x\max(a,b) + (1-x)\min(a,b)$. Where a and b are the input values of the nodes that will be operated and in the case of operators WA and OWA, x will be a random value within the range $[0,1]$. The Figure 3 shows the representation of a genotype and the equivalent tree, for a 2-dimensional 2-class dataset. In order to obtain the genotype that will represent the tree, CGP has at its disposal all the operators and fuzzy terms. Depending on the gene used as input of function genes, different trees can be achieved.

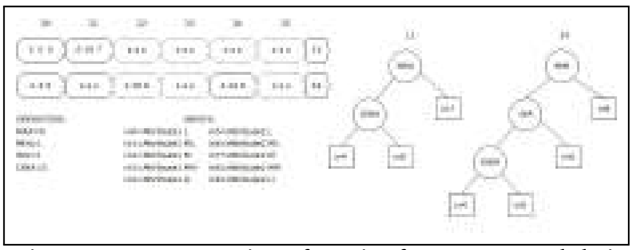


Figure 3. Representation of a pair of genotypes and their trees.

4.2 Evaluation The Fitness function is composed of two parts and can be seen in Equation 1. The first part deals with the purpose of obtaining a high accuracy and it is calculated using root mean square error (RMSE).

$$Fitness = w_1 * AP_1 + w_2 * AP_2 \quad (36)$$

$$RMSE = \frac{1}{N}\sqrt{\sum_{i=1}^{N}(o_i - d_i)^2} \quad (37)$$

$$AP_1 = Similarity = 1 - RMSE \quad (38)$$

The error is defined as the difference between value obtained in the output of the tree (o_i) and the target value (d_i), which must be 1 if the values presented in the inputs belong to the class represented by the tree and 0 otherwise. After the computation of RMSE, it is subtracted from 1, resulting in performance measure called $Similarity$. The second installment of the evaluation of genotypes is described by the Equation 4 and it is related to the aim of obtaining more interpretable trees.

$$AP_2 = \frac{Qgenes - ActiveGenes}{Qgenes} \quad (39)$$

As one can recognize, the value of AP_2 is increased as the number of active genes declines. Therefore, smaller trees have a higher value of AP_2 value than the larger ones. Consequently, it can guide the CGP search to favor smaller trees, which are easier to explain than the large ones.

4.3 Stopping Criteria In the CGP-FPT, there are three different stop criteria. The first one is related to the number of generations of the evolutionary process. When it reaches a predefined value, the evolution is stopped and the best solution so far is reported. The second criterion is triggered if there is no significant fitness improvement within a certain number of past generations. Finally, the third criterion was defined to avoid the model over fitting and it uses a standard cross validation procedure. It is fired if fitness calculated in the validation set starts to decrease, while the fitness calculated in the training set is still increasing. This situation indicates that the obtained model is losing the generalization capability and therefore the evolution must be ended. When it is not possible to separate a validation set from the original dataset, it is possible syn-thesize it artificially from the estimated probability density function obtained in the original dataset [10].

5. Results

This section presents a number of experimental studies to assess the performance of the CGP-FPT on several databases presented in Table 1 and gathered from the UCI machine learning repository. They were also used in [6], therefore, it is possible to contrast the FPTs induced with the proposed method with the ones obtained in [6]. In addition, these studies introduce a comparison with some other well-known classifiers, such as: Support Vector Machine with Linear Kernel (SVM-L) [11], the K-nearest neighbors (KNN) [10], the Random Forest (RF) [12] and Support Vector Machine (SVM-R) with Radial Basis Function Kernel [11]. The confrontation criterion was the estimation of the generalization performance measure accomplished through a cross-validation with 10 folds and 5 repetitions. As a matter of fact, two performance measures were used: the classification accuracy and the area under the ROC curve (AUC). However, the AUC provides a more robust measure than accuracy to evaluate classification models. Unfortunately, it can only be applied in binary classification problems, therefore, when the database has more than two classes, the AUC provided is given by the mean of the estimated AUCs in mode "one against all". The parameters of the others classifiers were obtained as follows: the number of neighbors in KNN was set at 1; the number of trees that are generated in Random Forest was equal to 50 and the number of attributes under which is made the partition was equal to 1.

Database	Instances	Attributes	Classes
Iris	150	4	3
Wine	178	13	3
Sonar	208	60	2
Pima	768	8	2
Balance	625	4	3
Habeman	306	3	2
Lupus	87	3	2
Breast_Cancer	683	9	2
Australian	690	14	2
Lawsuit	264	4	2
Ionosphere	351	33	2
Bupa	345	6	2
Transfusion	748	4	2

Table 1. Datasets.

The parameters of SVM were determined using nested cross-validation [13] and it is regarded as a way to avoid over fitting. It is called nested cross-validation because two cross-validations take place. The first one is called external cross-validation and it is used to provide an estimation of the generalization performance. The dataset is divided in 10 folds; therefore, the training procedure is done 10 times (for each repetition) using the chosen fold as a test set and the remaining folds as a training set. In order to find the best parameters, a second 5-fold cross-validation (also called internal cross-validation) was performed using only the training set. It means that the training is divided in five folds and the best parameters were chosen based on the performance in this internal cross-validation. Once the best parameters were found, the classifier was trained with the training set and its performance is evaluated on the test fold. The procedure used to find the best parameters was not biased since the test fold in the external cross-validation was not used to tune the parameters. The Table 2 presents comparison results.

Two variants of the CGP-FPT were used: CGP-FPT1 has 400 genotypes and evolved in 600 generations. The CGP-FPT2 has 50 genotypes evolved in 1250 generations. In this two variants, $w_1 = 0.7$ and $w_2 = 0.3$. Table 2 also presents PTTDE.25, which is the best accuracy result in FPT previous implementations [6]. These results were obtained from the java implementation of the Top-Down algorithm provided by authors [6]. Only the accuracy results were available. The analysis of Table 2 has shown that CGP-FPT2 presented competitive results. Regarding accuracy, it presented the best result in 3 datasets, accuracy equal or greater that 90% in 5 datasets and an absolute difference in accuracy from the best result less than or equal to 5% in 11 datasets. When one looks at the AUC, it can be seen that it obtained the best results in 5 datasets, AUC equal or greater than 0.9 in 5 datasets and absolute difference in AUC from the best result less than or equal to 5% in 10 datasets.

Also, to confirm these findings, Friedman's test and Nemenyi post-hoc test were applied. They are utilized to compare the performance of the classifiers and to verify if it is possible to identify a single classifier that presents a statistically significant better performance (at a significance level given by the $p-value = 0.05$) when one takes in account all the different datasets of the experiments. When they were executed on the accuracy measures, the tests revealed no statistically significant difference between the classifiers and the same has occurred in relation to the AUC. The second comparison was made with respect to the average tree depth between the CGP-FPT2 and PTTDE.25. This comparison was made for the worst case of CGP-FPT2, where the weight of the fitness function which favors smaller trees was set to zero, $w_1 = 1$ and $w_2 = 0$. From the results shown in Table 2, also confirmed by a Friedman test, it can be concluded that the CGP-FPT2 has better performance than the PTTDE.25 with respect to the average depth of the trees, since it generates trees with smaller depths. Possibly, this is a consequence of the beam-search on Top-Down strategy of PTTDE.25. As a result of its "greedy" characteristic, it chooses the best subtree candidates without going back after later and it dismisses candidates who may not have performed well individually but would have a better performance in a combination with a subtree chosen afterwards. Figure 4 presents the trees created by using the proposed model and the top-down strategy. It should be noticed that the tree in the Figure 4 (a) is smaller than the one shown in the Figure 4(b).

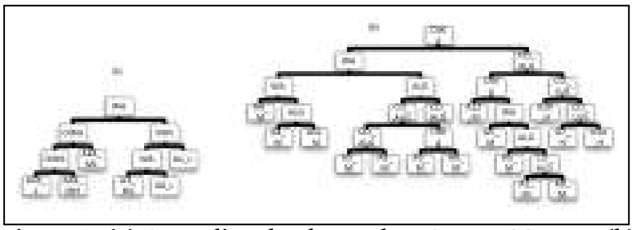


Figure 4. (a) Australian database, class 1 tree. CGP-FPT. (b) Australian database, class 1 tree. Top-Down strategy [6]

5. Conclusion

This paper proposed an alternative way to synthesize Fuzzy pattern trees, with Cartesian genetic programming as learning algorithm. The obtained results has shown that the performance of Fuzzy Pattern trees in the task of classification can approach the performance of some of the best classifiers available, but also providing a model that can be interpretable. The knowledge obtained in learning can be extracted from the model. Fuzzy Pattern trees constitute a viable alternative to the classical fuzzy rules-based models, because its hierarchical structure allows more compact representation and a compromise between accuracy and simplicity of the model.

Acknowledgments.The authors wish to thank R.Senge and E. Hullermeier for making the source code of their algorithm available.

Datasets	SVM-L		kNN		RF		SVM-R		CGP-FPT1		CGP-FPT2		PTTDE.25*	CGP-FPT2	PTTDE.25
	ACC	AUC	ACC	AUC	ACC	AUC	ACC	AUC	ACC	AUC	ACC	AUC	ACC	AD	AD
Iris	**0.96**	0.93	0.94	0.98	0.95	0.99	0.95	0.99	**0.96**	0.99	**0.96**	**1.00**	0.95	**2.60**	3.33
Wine	**0.98**	**1.00**	0.95	0.98	**0.98**	**1.00**	**0.98**	**1.00**	0.91	0.99	0.93	0.99	**0.98**	**2.33**	10.0
Sonar	0.80	0.89	0.87	0.95	0.88	0.94	**0.91**	**0.98**	0.77	0.85	0.77	0.84	0.80	**1.50**	10.0
Pima	**0.77**	**0.83**	0.70	0.72	**0.77**	**0.82**	0.71	0.79	0.73	0.79	0.75	0.77	0.76	**2.00**	4.00
Balance	0.87	0.93	0.77	0.88	0.85	0.95	**0.95**	**1.00**	0.83	0.91	0.84	0.94	0.87	4.30	**1.66**
Habeman	0.72	0.70	0.67	0.62	0.65	0.57	0.71	0.63	0.75	0.67	**0.76**	**0.75**	0.74	6.00	**2.00**
Breast_Cancer	**0.97**	**0.99**	0.95	**0.99**	0.96	**0.99**	0.96	0.98	0.96	**0.99**	0.96	**0.99**	0.96	**2.75**	5.00
Australian	**0.86**	**0.93**	0.80	0.86	0.79	0.92	0.85	0.91	0.85	0.91	0.84	0.89	0.85	**2.00**	5.00
Ionosphere	0.87	0.87	0.86	0.97	0.90	0.97	**0.92**	**0.98**	0.88	0.92	0.90	0.90	0.91	**2.00**	14.0
Lupus	0.74	0.86	0.68	0.67	0.62	0.72	0.73	0.78	0.73	0.84	0.75	**0.89**	**0.77**	4.75	**4.00**
Bupa	0.67	0.71	0.61	0.64	0.66	0.72	0.67	**0.73**	0.68	0.68	0.65	**0.73**	**0.70**	**4.25**	10.0
Transfusion	0.76	**0.74**	0.71	0.60	0.70	0.64	0.73	0.66	**0.77**	0.72	**0.77**	0.70	**0.77**	**4.75**	5.00
Lawsuit	**0.99**	**1.00**	0.96	0.97	0.97	0.99	0.96	0.99	0.96	0.99	0.96	0.99	0.94	**4.50**	9.00

Table 2. Results - Accuracy, AUC and Average Depth (AD).

References

[1] Witten, I.H. and Eibe F.: Data Mining: Practical Machine Learning Tools and Techniques, Second Edition (Morgan Kaufmann Series in Data Management Systems). San Francisco, CA, USA: Morgan Kaufmann Publishers Inc. (2005).

[2] Cordón, O.: A historical review of evolutionary learning methods for Mamdani-type fuzzy rule-based systems: Designing interpretable genetic fuzzy systems. International Journal of Approximate Reasoning, vol.52, no. 6 ; pp 894-913. (2011).

[3] Herrera,F.: Genetic fuzzy systems: taxonomy, current research trends and prospects, Evol. Intell., vol. 1, no. 1, pp. 27-46. (2008).

[4] Hullermeier, E.: Fuzzy Methods in Machine Learning and Data Mining: Status and Prospects, Fuzzy Sets Systems, vol. 156, no. 3, pp. 387-406. (2005).

[5] Huang, Z. ,Gedeon D. and Nikravesh M.: Pattern Trees Induction: A New Machine Learning Method, IEEE Trans. Fuzzy Syst., vol. 16, no. 4, pp. 958-970. (2008).

[6] Senge, R. and Hullermeier, E.: Top-Down Induction of Fuzzy Pattern Trees, Trans Fuz Sys, vol. 19, no. 2, pp. 241-252. (2011).

[7] Miller, J. F.: Cartesian Genetic Programming, 2011 edition. Heidelberg: New York: Springer. (2011).

[8] Miller, J. F:. What bloat? Cartesian Genetic Programming on Boolean problems, in 2001 Genetic and Evolutionary Computation Conference Late Breaking Papers, pp. 295-302. (2001).

[9] Miller, J. F. and Smith, S.L.: Redundancy and computational efficiency in Cartesian genetic programming, IEEE Trans. Evol. Comput., vol. 10, no. 2, pp. 167-174. (2006).

[10] Webb, A.R.R.: Density Estimation - Nonparametric, in Statistical Pattern Recognition, John Wiley & Sons, Ltd, pp. 81-122. (2002).

[11] Vapnik, V.N.: The Nature of Statistical Learning Theory, 2nd edition. New York: Springer. (2000).

[12] Breiman, L.: Random Forests, Mach. Learn., vol. 45, no. 1, pp. 5-32. (2001).

[13] Gavin, C.C, Talbot, N.L.C.: On Over-fitting in Model Selection and Subsequent Selection Bias in Performance Evaluation, J Mach Learn Res, vol. 11, pp. 2079-2107. (2010).

NEWS

Ph.D. Thesis defended by Ivana Micić

Faculty of Sciences and Mathematics, University of Niš, Niš, Serbia

Ivana Micić defended her PhD Thesis, entitled “Bisimulations for fuzzy automata”, on May 21, 2014. Her advisor was Dr. Jelena Ignjatović, from the University of Niš.

Bisimulations have been widely used in many areas of computer science to model equivalence between various systems, and to reduce the number of states of these systems.

One of the most important problems of the automata theory is to determine whether two given automata are equivalent, which usually means to determine whether their behavior is identical. In the context of deterministic, nondeterministic or fuzzy automata the behavior of an automaton is understood to be the language (fuzzy language) that is recognized by it, and two automata are considered equivalent, or more precisely language-equivalent, if they recognize the same language. For deterministic finite automata the equivalence problem is solvable in polynomial time, but for nondeterministic and fuzzy finite automata it is computationally hard.

The main aim of this thesis is studying of bisimulations for fuzzy automata, with the special emphasis on the problem of finding the greatest simulations and bisimulations. Besides, bisimulations are considered as a means for approximating the language-equivalence, as well as for use in the reduction of the number of states of fuzzy automata. The candidate considers fuzzy automata with membership values in complete residuated lattices and defines the notions of the left (right) derivative of the fuzzy language and minimal automaton of the given fuzzy language. Further, she defines crisp-deterministic fuzzy automata and the Nerode fuzzy automaton as an significant member of the class of all crisp-deterministic fuzzy automata, gives the definition of the minimal crisp-deterministic fuzzy automaton of the fuzzy language and representes the derivative automaton as an exponent of this class and observes different types of simulations and bisimulations, which play a significant role in the theory of bisimulations for fuzzy automata over complete residuated lattices. Then, for any of the mentioned types of simulations/bisimulations is provided an efficient algorithm for deciding whether there is a simulation/bisimulation of this type between the given fuzzy automata, and for computing the greatest one, whenever it exists. The candidate determines sufficient conditions under which this algorithm terminates in a finite number of steps, as well as sufficient conditions under which the infimum of this sequence is exactly the required fuzzy relation. Modifying these algorithms the algorithms for computing the greatest crisp simulations and bisimulations between fuzzy automata (which always terminate in a finite number of steps) are provided.

Afterwards, bisimulations are examined as a means for approximating the language equivalence between the fuzzy automata. The problem of expressing the language equivalence in terms of relationships between states of the given automata is very complicated in the case of nondeterministic and fuzzy automata.

Although, the bisimulations are shown to be a very good means for approximating the language equivalence between two fuzzy automata, there exist fuzzy automata which are language equivalent but there is no one type of bisimulation between them. In order to more precisely describe the class of all relations between the states of fuzzy automata, which preserve the language equivalence, the more general classes of bisimulations are introduced. The fundamental goal is the definition of two new kinds of bisimulations, weak forward and weak backward bisimulations, which provide better approximations of the language equivalence than forward and backward bisimulations. Moreover the weak simulations and bisimulations provide better results in the reduction of the states of fuzzy automata. It is important to mention, that weak forward (backward) simulation is generalization of the notion of forward (backward) simulation, respectively. It is shown that the existence of weak forward (backward) simulation between two automata implies language inclusion between them. In the class of all weak forward bisimulations from a fuzzy automaton into itself, the special attention is dedicated to these which are equivalences and it is shown that the reduction of fuzzy automata by means of the greatest weak forward bisimulation equivalence gives better results than one obtained by means of the greatest forward bisimulation equivalence.

Since fuzzy finite automata are generalizations of NFAs, in the work with fuzzy automata, the analogue minimization and reduction problems are also presented. Various researches considered the state reduction problem for fuzzy automata and they provided several algorithms which are also based on the idea for computing and merging indistinguishable states. It is worth mentioning, that although these algorithm are called the minimization algorithms, in the general case they do not produce the minimal fuzzy automaton in the set of all fuzzy automata recognizing a given fuzzy language, so these are just reduction algorithms. The candidate studies the reduction of fuzzy automata by means of right and left invariant fuzzy quasi-orders and right and left invariant fuzzy equivalences and proposes the new algorithm for computing the greatest right invariant fuzzy quasi-order on the given fuzzy automaton, which is based on the famous Paige-Tarjan’s coarsest partition problem. Afterwards, she presents the modification of previous algorithm, which computes the greatest right invariant equivalence on the given nondeterministic automaton.

The candidate has published her results in the leading international journals IEEE Transactions on Fuzzy Systems, Fuzzy Sets and Systems and Information Sciences.

Author's correspondece addres:
Ivana Micić
Višegradska 33
18000 Niš
Serbia
Email: ivanajancic84@gmail.com

NEWS

Ph.D. Thesis defended by Zorana Jančić

Faculty of Sciences and Mathematics, University of Niš, Niš, Serbia

Zorana Jančić defended her PhD Thesis, entitled "Bisimulations for fuzzy automata", on May 21, 2014. Her advisor was Dr. Jelena Ignjatović, from the University of Niš.

Many practical applications of automata require determinization, the procedure of converting a given nondeterministic automaton into an equivalent deterministic automaton. Numerous determinization algorithms have been developed and the standard one is the subset construction. In the case of fuzzy or weighted automata, the analogue of the subset construction can even yield an infinite crisp-deterministic automaton.

This PhD thesis provides several new determinization methods for fuzzy and weighted automata which give better results than all previously known determinization methods and even when all earlier methods produce infinite automata, the methods developed here can produce finite ones. The candidate provides a new algorithm for determinization of weighted finite automata over strong bimonoids which generates a crisp-deterministic weighted automaton equivalent to the original one always smaller than any other known determinization algorithm for weighted finite automata over stong bimonoids, as well as than any determinization algorithm for weighted finite automata over semirings or over lattice-ordered monoids.

Particularly important determinization methods are the canonization methods which result in a minimal crisp-deterministic fuzzy automaton language-equivalent to the origunal one. The best known canonization method for nondeterministic automata is the Brzozowski's double reversal algorithm, and here, the Brzozowski's algorithm is adapted to the fuzzy framework. It is shown that the fuzzy version of Brzozowski's algorithm outperforms all previous determinization methods for fuzzy automata. The candidate also provides another canonization algorithm based on the the degrees of inclusion of particular fuzzy languages which is even faster than the Brzozowski type algorithm.

A very important issue is to find such methods which will mitigate the potential enormous growth of the number of states during the determinization. The candidate proposes a somewhat different approach and provides two-in-one procedures that simultaneously perform determinization and state reduction. Subsequently, the candidate develops algorithms that produce smaller automata than all previous determinization algorithms while require the same computation time. The metodology based on simultaneous determinization and state reduction can also be used to improve performances of the proposed canonization algorithms.

The algorithms for computing the greatest simulations and bisimulations of all considered types have been implemented in the programming language C, and codes of these programs are shown in Appendix A.

The candidate has published her results in the leading international journals IEEE Transactions on Fuzzy Systems, Fuzzy Sets and Systems and Information Sciences.

Author's correspondece addres:
Zorana Jančić
Višegradska 33
18000 Niš
Serbia
Email: zoranajancic329@gmail.com

NEWS

Ph.D. Thesis defended by Pavel Vlašánek

Institute for Research and Application of Fuzzy Modeling, Centre of Excellence IT4Innovations division of the University of Ostrava, Ostrava, Czech Republic

Pavel Vlašánek defended his PhD Thesis, entitled "Inpainting method based on the F-transform", on 2014. His advisor was Dr. Irina Perfilieva, from the University of Ostrava.

The thesis is a contribution to the field of Image Processing. It deals with the problem of image restoration (reconstruction) that is regarded as one of the most complicated tasks. Its complexity comes from the fact that various image processing techniques are involved, e.g., up-scaling, interpolation, edge detection, inpainting, sharpening, etc. It should be emphasized that if interpolation is involved, then the complexity of the corresponding approach is high. Therefore, other technique than that based on interpolation was proposed and justified in the Thesis. The main contribution consists in solving the reconstruction problem using the F(fuzzy)-transform. Dr. Vlašánek developed several algorithms for image reconstruction and proved their effectiveness by comparing them with other standard techniques (based on various interpolation and sophisticated inpainting methods). The effectiveness was considered from two points of view: image quality and complexity of processing. He also showed how it is possible to enhance quality of the reconstructed image with the help of edge detection. Moreover, the latter method was also realized using the first-degree F-transform. Dr. Vlašánek made several experiments in order to choose optimal setting of the F-transform parameters. It should be emphasized that the successful application of the F-transform method to the problem of image reconstruction was not evident at the beginning. Dr. Vlašánek took the problem as a challenge and found a totally new innovative and effective solution.

NEWS

Ph.D. Thesis defended by Petra Hod'áková

Institute for Research and Application of Fuzzy Modeling, Centre of Excellence IT4Innovations division of the University of Ostrava, Ostrava, Czech Republic

Petra Hod'áková defended her PhD Thesis, entitled "Fuzzy(F-)transform of functions of two variables and its applications in image processing", on 2014. Her advisor was Dr. Irina Perfilieva, from the University of Ostrava.

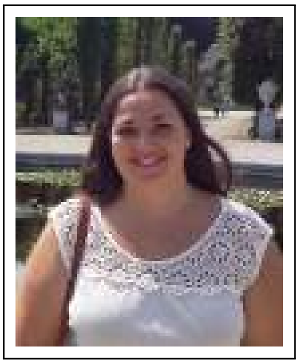
The thesis is devoted to the theory and applications of fuzzy transform (F-transform), both have been extensively developed in last years. The general aim of the thesis was to develop the F-transform technique for functions of two variables on the basis of orthogonal projections on special Hilbert spaces and to apply some of the obtained results to the edge detection problem in image processing. Another goal is to extend the ordinary F-transform to the F-transform of a higher degree (F^s-transform, $s \geq 0$) with the purpose to estimate approximations of partial derivatives. The theoretical background of the work assumes deep knowledge in functional analysis, numerical methods and integral transforms, it requires good vision of their synergy.

It has been shown that the elaborated theory can be successfully used in the image processing and especially in the problem of edge detection, which is an important constituent of a many complex problems in the field. Together with other colleagues, Dr. Hod'áková designed edge detection algorithms and made comparison of their effectiveness. The dissertation thesis is an example of high quality text, which can be used in theoretical foundations of the soft computing as well as in its important applications.

NEWS

Ph.D. Thesis defended by María Eugenia Cornejo

Department of Mathematics, University of Cádiz, Cádiz, Spain



María Eugenia Cornejo defended her PhD Thesis, entitled "Adjoint operators in general frameworks and their applications", on May 11, 2015. Her advisor was Dr. Jesús Medina, from the University of Cádiz.

Adjoint triples arise as a generalization of a t-norm and its residuated implication. They are basic operators to make the calculus in multi-adjoint logic programming, multi-adjoint concept lattices, multi-adjoint fuzzy rough sets and multi-adjoint fuzzy relation equations, providing more flexibility and increasing the range of applications in the setting in which they are considered.

This thesis is focused on the study of the adjoint triples, their properties and applications. Firstly, important properties of adjoint pairs/triples and the algebraic structures associated with these operators, which are called multi-adjoint algebras, have been presented.

Later, this work presents an intense comparison among different general algebraic structures such as implication triples, sup-preserving aggregations, quantales, u-norms, uninorms and general implications considered in extended-order algebras. This comparative study proves that the use of these algebraic structures, in environments requiring residuated implications, provides particular cases of multi-adjoint algebras.

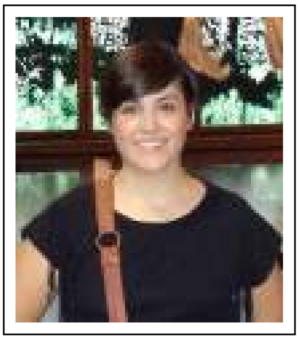
Moreover, adjoint negations are introduced as a new generalization of residuated negations that satisfy the most significant properties. Besides generalizing this kind of negations, this work shows that adjoint negations generalize, at least, three of the negation operators most useful in the literature, such as the negation operators introduced by Trillas, the pairs of weak negations presented by Georgescu and Popescu and the negation operators defined by Della Stella and Guido in the setting of a specific extended-order algebra.

The last part of the thesis describes a process of how to represent a multi-adjoint logic programming as a multi-adjoint relation equation, which is important in order to use these fuzzy relation equations as a decision support system for fuzzy logic. The solvability of these equations is provided from the theory of Fuzzy Formal Concept Analysis.

NEWS

Ph.D. Thesis defended by Eloísa Ramírez

Department of Mathematics, University of Cádiz, Cádiz, Spain



Eloísa Ramírez defended her PhD Thesis, entitled "Multilattices and attribute reduction in multi-adjoint concept lattices", on May 12, 2015. Her advisor was Dr. Jesús Medina, from the University of Cádiz.

Since its introduction in the eighties by B. Ganter and R. Wille, Formal Concept Analysis has become an appealing research topic. It is a theory of data analysis which identifies conceptual structures among data sets. Specifically, it is a tool for extracting pieces of information from databases that contain a set of attributes A and a set of objects B together with a relationship between them. These pieces of information are called concepts and they can be hierarchized to obtain concept lattices.

Attribute reduction is a very important part in Formal Concept Analysis because the difficulty in building the concept lattice increases exponentially when the number of objects and attributes increases. Therefore, one of the most important goals in this theory is the reduction of the context, removing the irrelevant information. Moreover, real databases usually give rise to complex concept lattices, from which extracting conclusions can be a really difficult task. Consequently, another important issue is the reduction of the size of the original concept lattice. This thesis has been focused on both these goals. Firstly, it introduces several results in order to classify the set of attributes. From this classification a mechanism to reduce the context on a fuzzy environment is obtained, which generalizes the current existing procedures. The most innovative aspect related to this contribution is that it maintains all the knowledge of the relational system.In addition, two procedures to reduce the size of a multi-adjoint concept lattice are presented. One of them considers thresholds in the concept-forming operators and this reduction method generalizes existing mechanisms based on this philosophy. Another procedure introduced shows a reduction from the irreducible elements of the lattice. This one provides an interesting property, that is, the reduced concept lattice is a sublattice of the original one and, consequently, the use of this mechanism does not involve the loss or modification of the original information.Lastly, the thesis concludes by demonstrating an extension of the theory of Formal Concept Analysis based on multilattices. As a consequence, the range of applications of Formal Concept Analysis has been increased.

NEWS

Ph.D. Thesis defended by Marco Lucarelli

Department of Informatics, University of Bari A. Moro, Bari, Italy

Marco Lucarelli defended his PhD Thesis, entitled “DC*: an Algorithm for Automatic Acquisition of Interpretable Fuzzy Information Granules”. His advisor was Dr. Corrado Mencar, from the University of Bari A. Moro.

Several real world problems require more than just accurate solutions. In many cases, users (physicians, managers, etc.) have to be convinced about the reliability of the knowledge base, and hence they may be interested in systems capable to offer good support in terms of both accuracy and comprehensibility of the knowledge base. When intelligent systems are used to acquire knowledge from data, a methodology is required to derive interpretable knowledge that final users can easily understand. Fuzzy rule-based systems (FRBSs) are tools that enable knowledge representation and inference through fuzzy rules denoted by linguistic terms. The main point of strength of FRBSs is the possibility of establishing a semantic similarity (or co-intension) between the fuzzy sets that are used in their rules and the implicit semantics of the linguistic terms that are used to denote them. In this way the users of a FRBS can read and understand fuzzy rules, as well as revise and integrate rules with domain knowledge. In other words, the FRBS can be interpretable for users. However, when FRBSs are acquired from data through some learning scheme, the semantic co-intension between fuzzy sets and linguistic meanings is often lost. This happens because fuzzy sets are usually shaped in order to optimize a specific performance measure, usually defined in terms of accuracy error. Nevertheless, the loss of semantic co-intension in a rule base determines a FRBS that is no longer interpretable.

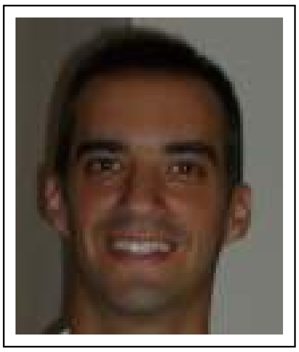
The development of specific learning algorithms is intended to overcome the interpretability loss in the process of acquiring knowledge from data. Mainly, these learning schemes drive the adaption process so that a number of interpretability constraints is satisfied. Many learning algorithms for acquiring interpretable models require to fix the granularity of fuzzy partitions, i.e. the number of fuzzy sets that partition each input feature: the aim of such algorithms is to find the best shapes of the fuzzy sets in the partition so as to optimally balance accuracy and interpretability of the final system. Moreover, human interaction is required in the model building process as well as in the final model choice. As a result, in many cases a trial-and-error approach is used to select the best granularity for each feature (in terms of model accuracy) without looking at the granule characteristics (number, shape, position, etc.) which usually affect the model interpretability. As a matter of fact, the optimal number of fuzzy sets per feature is often unknown, also could be different for each feature and is strictly problem-dependent. Moreover, the granularity of a solution should be changeable taking into account the user needs, the context and the model complexity.

The lack of such methods is filled by the proposed approach named DC*, derived from the Double Clustering framework. The key feature of DC* is its ability of providing an automatic interpretable fuzzy granulation of classified data, exploiting hidden relationships among data, and thus discovering the optimal granularity level for each problem feature. Then, the obtained partition can be translated in a highly interpretable fuzzy rule base. It is worth noting that the whole process requires the definition of only one hyper-parameter representing the maximum granularity level of the final fuzzy rule base - i.e. the maximum number of rules; such parameter is easily understandable and configurable by the user. DC* is composed by three phases: the first accomplishes a compression of data that identifies a number of prototypes over the problem space. Then DC* clusters the prototype projections on all dimensions simultaneously: in this way it is possible to minimize the number of clusters for each feature. This is accomplished through an in-formed search procedure based on the A* algorithm. The resulting one-dimensional clusters provide information to define fuzzy partitions that satisfy a number of interpretability constraints and exhibit variable granularity levels. The fuzzy sets in each partition can be therefore labeled by meaningful linguistic terms and used to represent knowledge in a natural language form. Experimental results on benchmark datasets highlight the DC* peculiarities compared with other algorithms in terms of interpretability/accuracy trade-off as well as its efficiency in terms of resources required in the granulation process.

NEWS

Ph.D. Thesis defended by David García Muñoz

University of Granada, Granada, Spain



David García Muñoz defended his PhD Thesis, entitled "Extensions of the genetic iterative approach for learning fuzzy rules". His advisors were Dr. Antonio González Muñoz and Dr. Francisco G. Raúl Pérez Rodríguez from the University of Granada.

The dissertation makes sense in the framework in which it is necessary to handle large amounts of data, some of them including vague or imprecise information.

Particularly, the methods here proposed are based on the Iterative Rule Learning (IRL) approach, characterized by the use of a sequential covering strategy together with a genetic algorithm in order to learn fuzzy rules.

Considering this fact, we focus our efforts in two main objectives: on the one hand to consider the indirect relevance of the input attributes in the learning process and on the other hand to improve the IRL model used by a basic fuzzy rule-based learning algorithm (NSLV) to be able to iteratively review the learned knowledge.

In this sense, the first objective is achieved through the use of feature construction techniques which allows the extraction of additional information resulting from the combination between the original variables. So, the learning algorithm handles not only the information given by the input variables, but also that given by the new constructed features. Following this idea, in this work we have presented three methods including feature construction techniques: one of them using relations in the antecedent of fuzzy rules (NSLV-R), another one using functions in the antecedent of such rules (NSLV-F) and the last one combining both relations and functions (NSLV-FR).

It was experimentally proved that feature construction techniques work well when looking for accurate models, with an interpretability level nearer to natural language. Nevertheless, some well-known interpretability measures refers to the number of rules of rule bases and the number of conditions per rule (also per rule base), as key elements in order to consider a ruleset as interpretable.

It is inside this searching process of interpretable rule bases where the second main objective takes place. The ability of a fuzzy rule-based learning algorithm to review the knowledge as part of its own learning process, allows tuning the knowledge in each step. In this way, our proposal including knowledge review (NSLV-AR), decides in each iteration whether to replace or not a previously learned set of rules.

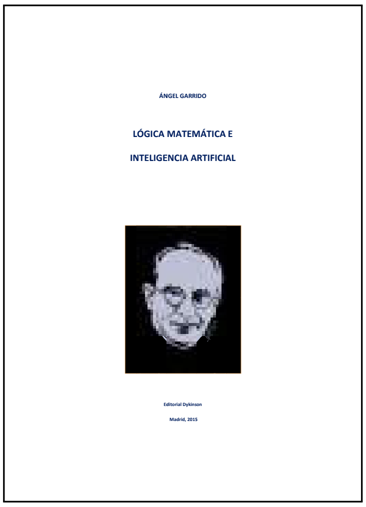
Finally, the last proposal (SLAVE3), arises from the need to integrate the ideas previously exposed into a new model achieving a good trade-off between accuracy and interpretability. So, on the one hand we were looking for an algorithm with a high level of accuracy, similar to those using feature construction, and on the other hand, which also were able to improve the interpretability (by reducing the complexity at a rule/rule base level), when compared with those last ones.

All the algorithms previously mentioned demonstrate their performance through an exhaustive experimental study.

NEWS

New book by Angel Garrido

Angel Garrido
LÓGICA MATEMÁTICA E INTELIGENCIE ARTIFICIAL
Editorial Dykinson
Madrid, 2015



This book aims to fill a certain void in the day that we see on these issues. Therefore, we have discussed here some of the other great logic; including Bernhard Bolzano, Franz Brentano, Twardowski Kazimierz, Stanislaw Lesniewski, the Lvov-Warsaw School (ELV), in contrast with the more "mediatic" or commented Vienna Circle (Wiener Kreis). His predecessors, as were Leibniz, or the aforementioned Bernardo Bolzano and Franz Brentano. His disciples, Edmund Husserl and Kazimierz Twardowski, a very brilliant group: Jan Lukasiewicz, Alfred Tarski, or Stefan Banach. And then, we should refer to Charles Sanders Peirce, David Hilbert, Ernst Zermelo. We will also consider other fundamental people: Bertrand Russell, Alfred North Whitehead and Ludwig Wittgenstein. Without ceasing to mention Brouwer and his disciple Heyting. And try to Alonzo Church, S. C.Kleene, Lofti A. Zadeh, etc. Without thereby exhausting all possible fields and candidates worthy of consideration. We hope so complete that line more time, often fragmentary and incomplete, but always interesting.

Also we analyze the relation with the new problems proposed by the fields of Computer Science; in particular, on Artificial Intelligence.

Some of the information contained herein may be considered in some way (but in any case, very distantly) connected with the doctoral thesis, entitled *Philosophy and Mathematics of Vagueness and Uncertainty*. But even so, I do not think so broadly, all have been following very filtered, edited and expanded, always moving with the purpose of bringing such knowledge to a sector of the reading public as broad as possible, and in its day I asked him to do the members of the tribunal of that thesis, considering how little these themes are still known in Spain as well as in some other countries.

You can also find many analyzes related to these themes in two of my previous books, recently published an excellent yet economical edition, by Editorial Dykinson; these are the *LOGIC OF OUR TIME* (2014), and *APPLIED LOGIC. VAGUENESS AND UNCERTAINTY* (2014).

Efforts have been made that these are brief, yet self-contained and easy reading, and a very affordable price, no small matter when it comes to research. What is to be a work, first, it's interesting, then, to be well written. Following this line: to give new information so as to better know and appreciate certain great contributions to the history of logic (as yet insufficiently known, at least, not in our cultural and academic circles) and thought in general the author has written this book.

And recently, Ángel Garrido has received the First Extraordinary Award of Doctorate at the UNED.

NEWS

Weather forecasting in natural language using fuzzy technologies

The Galician (NW Spain) Weather Forecasting Agency, Meteogalicia (`www.meteogalicia.es`), has released on May 25th a public service providing natural language forecast automatically generated from data. This service is implemented by the GALiWeather system, a Data-To-Text solution that includes a content determination module involving computing with words. GALiWeather was developed at the Intelligent Systems Unit of the Research Centre in Information Technologies of the University of Santiago de Compostela -CiTIUS-. By using GALiWeather, Meteogalicia is now able to provide to the visitors of their official Website customized weather forecasts in natural language for each of the 314 Galician city councils. These forecasts were so far provided in graphical and numerical manner and therefore, were not easily understandable by the users.

GALiWeather is one of the research results of the R&D project “Linguistic description of complex phenomena: generalized fuzzy quantifiers in temporal propositions (QTEMP),” that was developed in the CiTIUS between 2012 and 2014. A team of four CiTIUS researchers leaded by Prof. Alberto Bugarín have collaborated with seven experts of Meteogalicia for the conception, requisites capture, design and implementation of this tool. QTEMP is a part of the coordinated project “Linguistic Description of Complex Phenomena” developed in cooperation between the CiTIUS and the European Centre for Soft Computing (ECSC), funded by the Spanish Ministry for Economy and Competitiveness.

So far, the textual forecasting Meteogalicia offered to its visitors was a single general description of the short-term weather trend in Galicia. After integrating GALiWeather in its public Website, Meteogalicia has expanded this service to provide specific natural-language predictions for each municipality. For practical and economic reasons, this would not be possible without the automation this intelligent system provides. Predictions are now automatically generated in natural language from the available data, just as a meteorologist would do, producing high-quality texts that are indistinguishable from the texts an expert meteorologist would produce.

GALiWeather generates predictions in natural language, employing numerical data, expert knowledge from the meteorologists and computing with words techniques (linguistic variables, fuzzy quantification and others) that allow, first, converting data into basic linguistic descriptions (content determination stage); a subsequent process converts these into a narrative in natural language (linguistic realization stage), expressed in Spanish and Galician (the supported languages by now). Narratives offer a four days weather forecast view, including information about the state of the sky (clouds and rain), wind and temperature, as well as Air Quality Index. The modelling of vague terms and expressions using the computing with words paradigm is integrated in GALiWeather with Natural Language Generation techniques for producing the final narratives at the Meteogalicia website, which received more than 55 million accesses last year.

More Scientific and technical details about GALiWeather have been recently published in [1].

References

[1] A. Ramos-Soto, A. Bugarín, S. Barro, J. Taboada. Linguistic Descriptions for Automatic Generation of Textual Short-Term Weather Forecasts on Real Prediction Data. IEEE Transactions on Fuzzy Systems, vol. 1, no. 23, pp. 44-57. 2015.